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# Insurance Markets with Interdependent Risks

Wing Yan Shiao

Submitted in Partial Fulfillment of the  
Prerequisite for Honors in Economics

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# Insurance Market with Interdependent Risks

Wing Yan Shiao

## Abstract

This paper investigates an insurance market with adverse selection, moral hazard and across-contract endogeneity, under monopoly and perfect competition. We characterize the equilibrium in a market without endogeneity and study how the introduction of across-contract endogeneity into the model distorts the optimal contracts. The across-contract endogeneity can be viewed as a second source of endogeneity, in addition to moral hazard, that further reduces insurance coverage if the insurer considers its implication when choosing contracts. We show that a monopolist internalizes the externality exerted by the contracts and offers contracts with less coverage, which induce a lower level of average risk. Competitive insurers fail to account for the interdependence of risks and do not adjust accordingly. They offer excessive insurance, which leads to a higher level of average risk and creates inefficiency. Our analysis suggests that there is a trade-off between monopoly and perfect competition.

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# 1 Introduction

Insurance markets have been of interest to economists because of asymmetric information. Rothschild and Stiglitz (1976) show that an equilibrium may not exist in a competitive insurance market with pure adverse selection when there are too few high-risk individuals in the market. Since then, there have been different attempts to understand the insurance market by modifying the equilibrium concept and introducing interactions between the insurers and the insured. Others have addressed the problem of moral hazard in an insurance market. Shavell (1979) discusses the optimal insurance coverage in a market with moral hazard and studies the condition for partial coverage to be provided. Arnott and Stiglitz (1986) show that differential commodity taxation can remedy the market failure caused by moral hazard.

This paper studies the insurance market with interdependent risks under monopoly and perfect competition. With across-contract endogeneity of risks, a contract offered by the insurer to one agent exerts an externality on other agents in the market. Specifically, we consider how the externality influences other agents' choice of self-protection, a concept introduced by Ehrlich and Becker (1972) that an individual can incur a cost to reduce the probability of loss while leaving the magnitude of loss unchanged. We show that under some natural assumptions, the monopoly market with endogeneity is less risky since a monopolist internalizes the externality, while the competitive market with endogeneity is inefficient because the average level of risk is too high.

Interdependence of risks is best exemplified by automobile insurance. In the presence of moral hazard, a driver's coverage affects her level of self-protection, which will affect the ambient level of risk and indirectly influence another driver's choice of accident probability. If a lower coverage causes one driver to drive with more precaution, then another driver will be exposed to a safer driving environment. As a result, she will put less effort into driving. Interdependence of risks is highly relevant to the real world, yet most existing literature abstracts from this endogeneity – to our knowledge, there is limited literature that focuses on the interaction between adverse selection and across-contract endogeneity of risks. Hofmann (2007) is the only closely related paper; she shows that a monopolist can achieve a socially optimal level of risk within a context of limited expected profits of the monopolist, compulsory insurance and a binary decision of self-protection.

We first investigate the optimal contracts designed by a monopolist facing two types of individuals, who only differ in the cost of self-protection, in a market without across-contract endogeneity. Both separating and pooling equilibria are found to be possible. In solving the monopolist's profit maximization problem with across-contract endogeneity, we adopt techniques from Rothschild and Scheuer (2013) and Chen and Rothschild (2015) – we decompose the optimization problem into an inner problem, which chooses the optimal contracts to maximize profits given a fixed level of average risk, and an outer

problem, which picks the profit-maximizing level of average risk in the market. By examining the first-order conditions of the Lagrangian of the inner and outer problem, we characterize the distortions on the equilibrium contracts caused by across-contract endogeneity. Under certain conditions, the monopolist will offer contracts that create a less risky environment relative to the market without across-contract endogeneity.

In the analysis of the competitive market without across-agent endogeneity, we find that both pooling and separating contracts are possible in the equilibrium. We show the competitive market to be inefficient when across-contract endogeneity is introduced to the market – the average risk level is too high as there exists a feasible allocation that makes both types better off if the average risk is lowered.

The results are suggestive of a trade-off between monopoly and perfect competition. A monopolist generally extracts rents due to her market power; our model implies that the competitive market is inefficient because of insurers' failure to take across-contract externality into account. Recall that under moral hazard, agents are offered partial coverage due to incentive problems. Moral hazard can be viewed as the first source of endogeneity in our model and across-contract endogeneity is the second. The latter further reduces the optimal insurance coverage in a monopoly market relative to the level of coverage in a monopoly market with only moral hazard. Nevertheless, competitive insurers do not consider the second type of endogeneity and offer contracts that induce an unnecessarily high level of average risk. Due to excessive insurance, the competitive market outcome is not constrained Pareto efficient.

The paper proceeds as follows. Section 2 describes the setup of the model, the equilibrium concepts considered in each market and the assumptions that we make in our analysis. Section 3 presents our analysis of the optimal contracts in a monopoly and competitive market, with and without across-contract endogeneity respectively. Section 4 discusses the implications of the results and concludes.

## 2 Setup

### 2.1 The Model

There are two types of individuals in the insurance market, low-risk types (denoted  $L$ ) and high-risk types (denoted  $H$ ), and  $\lambda \in [0, 1]$  is the fraction of  $L$  types in the market. The individuals have von Neumann-Morgenstern preferences with expected utility function  $u(x)$ . We assume that  $u'(x) > 0$  and  $u''(x) < 0$ .

Each type of individual has wealth  $W$  and suffers from a potential loss  $D$  with probability  $p$ , where  $p$  depends on the individual's efforts for self-protection, introduced by Ehrlich and Becker (1972).<sup>1</sup> Such efforts are costly and also dependent on type and the

---

<sup>1</sup>Note that Ehrlich and Becker consider the cost of self-protection in terms of additional expenditure

ambient level of risk – the individual will incur a cost of  $h(p, \bar{p}, k_i)$ , where  $k_i$  is the type-specific parameter, to maintain her personal risk at  $p$  when the average risk in the market is  $\bar{p}$ . We assume that  $\frac{\partial h}{\partial p} < 0$ ,  $\frac{\partial^2 h}{\partial p^2} > 0$ ,  $\frac{\partial h}{\partial \bar{p}} > 0$ ,  $\frac{\partial^2 h}{\partial \bar{p} \partial p} < 0$ ,  $\frac{\partial h}{\partial k} > 0$  and  $\frac{\partial^2 h}{\partial p \partial k} < 0$ .<sup>2</sup> In our analysis, we will suppress the argument  $k_i$  and write  $h(p, \bar{p}, k_i)$  as  $h_i(p, \bar{p})$ .

Insurance contracts are characterized by a non-negative premium  $R$  and a non-negative indemnity  $I \leq D$ . Equivalently, we can describe a contract by the consumptions in loss and no loss states,  $c_L$  and  $c_{NL}$  respectively, which satisfy  $c_{NL} \leq W$  and  $c_L \geq W - D$ . Graphically, the insurance contract lies to the northwest of the endowment point  $(W, W - D)$  in the  $(c_{NL}, c_L)$  space. We assume that people always want to buy insurance.

Given a contract  $(c_{NL}, c_L)$ , if a type- $i$  agent chooses a risk level  $p_i$ , she will get an expected utility of

$$Eu^i(c_{NL}, c_L; \bar{p}) = p_i u(c_L) + (1 - p_i) u(c_{NL}) - h_i(p_i, \bar{p}).$$

Define the indirect utility of type  $i$  to be

$$U^i(c_{NL}, c_L; \bar{p}) \equiv \max_{p_i} p_i u(c_L) + (1 - p_i) u(c_{NL}) - h_i(p_i, \bar{p})$$

and the maximizer to be

$$p_i^*(c_{NL}, c_L; \bar{p}) = \operatorname{argmax}_{p_i} Eu^i(c_{NL}, c_L; \bar{p}).$$

The insurer can observe neither the personal risk  $p_i^*$  nor the type  $i$ .

## 2.2 Equilibrium Concepts

The only nonstandard element in our model is the average risk in the market  $\bar{p} = \lambda p_L + (1 - \lambda) p_H$ , which we assume to capture the across-contract endogeneity by including  $\bar{p}$  in the indirect utility function and cost function as an argument. In a market with across-contract endogeneity, an insurance contract has an externality – the insurance coverage will influence one's risk level due to moral hazard and in turn the ambient level of risk, which has an effect on other individuals' choice of personal risk.

Towards studying the optimal contracts in a monopoly or competitive insurance market with across-contract endogeneity, it is helpful to look at markets without endogeneity.

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throughout both states whereas in our model, self-protection induces a utility cost but does not reduce wealth in either state.

<sup>2</sup>The derivative  $\frac{\partial^2 h}{\partial \bar{p} \partial p} < 0$  means that the cost to reduce risk at the margin is higher when  $\bar{p}$  is higher. This ensures that if  $\bar{p}$  increases, the risk level  $p_i$  chosen by an individual will increase for a given contract. The derivative  $\frac{\partial^2 h}{\partial p \partial k} < 0$  implies that  $H$  types bear higher costs at the margin to reduce their risks and therefore the high-risk types always choose higher risks than low-risk types at any point.



In this section, we discuss the different equilibrium concepts considered under different market structures. Without across-contract endogeneity, the ambient level of risk  $\bar{p}$  is thought of as fixed by both the insurer(s) and individuals. Thus  $\bar{p}$  can be treated as an exogenous parameter and for notational convenience we will suppress the argument in the functions and quantities dependent on  $\bar{p}$ . However, in the presence of such endogeneity, we will take  $\bar{p}$  into account and consider an additional consistency constraint in solving the problem.

### 2.2.1 Monopoly Market without Endogeneity

The monopolist wishes to maximize her total profit  $\pi = \lambda\pi_L + (1 - \lambda)\pi_H$  from offering insurance to both types, where  $\pi_i(c_{NL}, c_L) = -p_i^*(c_{NL}, c_L)c_L - (1 - p_i^*(c_{NL}, c_L))c_{NL} + W - p_i^*(c_{NL}, c_L)D$  is the profit from selling a contract to type  $i$ . Each type's contract should provide a utility no less than the type's outside option. The monopolist cannot observe an individual's type  $i$  or risk  $p_i^*$ . By the revelation principle (Myerson 1979), we can think of the monopoly's profit maximization problem as one of choosing the type-specific contracts  $(c_{NL}^L, c_L^L)$  and  $(c_{NL}^H, c_L^H)$  subject to type-specific individual rationality constraints and incentive compatibility constraints. Hence, the monopoly's profit maximization problem can be written as

$$\begin{aligned} \max_{c_{NL}^H, c_L^H, c_{NL}^L, c_L^L} \quad & \pi = \lambda\pi_L(c_{NL}^L, c_L^L) + (1 - \lambda)\pi_H(c_{NL}^H, c_L^H) & (1) \\ \text{subject to} \quad & U^L(c_{NL}^L, c_L^L) \geq \bar{U}^L & (IR_L) \\ & U^H(c_{NL}^H, c_L^H) \geq \bar{U}^H & (IR_H) \\ & U^L(c_{NL}^L, c_L^L) \geq U^L(c_{NL}^H, c_L^H) & (IC_L) \\ & U^H(c_{NL}^H, c_L^H) \geq U^H(c_{NL}^L, c_L^L), & (IC_H) \\ & \text{where } \bar{U}^i = U^i(W, W - D). \end{aligned}$$

Although the problem is framed as, without loss of generality, “choosing” contracts for both types, we do not impose an additional restriction that all types must purchase insurance. Indeed, the contract  $(W, W - D)$  corresponds to the endowment point where an agent does not purchase any insurance.

Specifically,  $(IR_L)$  and  $(IR_H)$  are the individual rationality constraints of the  $L$  type and the  $H$  type respectively. For a type- $i$  agent to accept the type  $i$ 's contract  $(c_{NL}^i, c_L^i)$ , she has to obtain a minimum utility of  $\bar{U}^i$ , the utility that she will get without purchasing insurance. These two constraints ensure the participation of both types in the market.  $(IC_L)$  and  $(IC_H)$  are the incentive compatibility constraints of the  $L$  type and the  $H$  type respectively. By accepting the type  $i$ 's contract, a type- $i$  agent will be no worse off than buying the type  $j$ 's contract. Therefore, each type has no incentive to mimic the

other type and deviate from her own type's contract.

### 2.2.2 Monopoly Market with Endogeneity

When across-agent endogeneity is present, in maximizing her profits, the monopolist recognizes the externality of the contract offered to one individual on others' cost of reducing personal risk, and thus she will take the endogeneity into account when designing the contracts. In the profit maximization problem, she chooses  $\bar{p}$  in addition to the  $H$ -type and  $L$ -type contracts, subject to the participation constraints, incentive compatibility constraints and an additional consistency constraint, which ensures the average risk level induced by the contracts chosen to be consistent with  $\bar{p}$ . The profit maximization problem can be stated formally as

$$\begin{aligned}
\max_{\bar{p}} \max_{c_{NL}^H, c_L^H, c_{NL}^L, c_L^L} \pi &= \lambda \pi_L(c_{NL}^L, c_L^L; \bar{p}) + (1 - \lambda) \pi_H(c_{NL}^H, c_L^H; \bar{p}) & (2) \\
\text{subject to } U^L(c_{NL}^L, c_L^L; \bar{p}) &\geq U^L(W, W - D; \bar{p}) & (IR_L) \\
U^H(c_{NL}^H, c_L^H; \bar{p}) &\geq U^H(W, W - D; \bar{p}) & (IR_H) \\
U^L(c_{NL}^L, c_L^L; \bar{p}) &\geq U^L(c_{NL}^H, c_L^H; \bar{p}) & (IC_L) \\
U^H(c_{NL}^H, c_L^H; \bar{p}) &\geq U^H(c_{NL}^L, c_L^L; \bar{p}). & (IC_H) \\
\lambda p_L^*(c_{NL}^L, c_L^L; \bar{p}) + (1 - \lambda) p_H^*(c_{NL}^H, c_L^H; \bar{p}) &= \bar{p}. & (\text{Consistency})
\end{aligned}$$

As in the monopoly's problem without endogeneity,  $(IR_L)$  and  $(IR_H)$  are the participation constraints of the  $L$  type and the  $H$  type respectively, while  $(IC_L)$  and  $(IC_H)$  are the incentive compatibility constraints of the  $L$  type and the  $H$  type respectively. Note that each type's outside option, indirect utility function and profit function all depend on  $\bar{p}$  due to across-contract endogeneity.

### 2.2.3 Competitive Market without Endogeneity

In the case of a competitive market, we consider the Miyazaki(1977)-Wilson(1977)-Spence(1978) (MWS) equilibrium concept. The insurers will "design" incentive compatible contracts to maximize the  $L$  type's utility while offering the  $H$  type a contract that gives at least their first-best utility and making an overall zero profit. Under MWS contracts, it is possible to observe cross-subsidization from the  $L$  type to the  $H$  type – in contrast to the Rothschild-Stiglitz contracts, which require each individual contract to yield zero profit, MWS contracts only require the insurer to make an overall zero profit.

Formally, the MWS contracts solve the following maximization problem:

$$\begin{aligned}
& \max_{c_{NL}^H, c_L^H, c_{NL}^L, c_L^L} U^L(c_{NL}^L, c_L^L; \bar{p}) & (3) \\
& \text{subject to } U^H(c_{NL}^H, c_L^H; \bar{p}) \geq \tilde{U}^H(\bar{p}) & (3.MU_H) \\
& U^H(c_{NL}^H, c_L^H; \bar{p}) \geq U^H(c_{NL}^L, c_L^L; \bar{p}) & (3.IC_H) \\
& U^L(c_{NL}^L, c_L^L; \bar{p}) \geq U^L(c_{NL}^H, c_L^H; \bar{p}) & (3.IC_L) \\
& \lambda \pi_L(c_{NL}^L, c_L^L; \bar{p}) + (1 - \lambda) \pi_H(c_{NL}^H, c_L^H; \bar{p}) = 0, & (3.\text{Zero profit})
\end{aligned}$$

where  $\tilde{U}^H(\bar{p})$  is the first-best utility of  $H$  types, i.e.,

$$\begin{aligned}
\tilde{U}^H(\bar{p}) &\equiv \max_{c_{NL}^H, c_L^H} U^H(c_{NL}^H, c_L^H; \bar{p}) \\
&\text{subject to } \pi_H(c_{NL}^H, c_L^H; \bar{p}) = 0.^3
\end{aligned}$$

Recall that  $\bar{p}$  is exogenous in this case. (3.MU<sub>H</sub>) is the participation constraint of  $H$  types. The  $H$ -type contract must yield a utility no less than the first-best utility of  $H$  types; otherwise, they can reveal their type to the insurer and purchase their first-best contract. (3.IC<sub>L</sub>) and (3.IC<sub>H</sub>) are the usual incentive compatibility constraints to ensure that each type self-selects into her own type's contract. The zero-profit constraint requires the insurer to break even.

It is worth noting that in principle the first-best allocations for both types may be achieved – Bond and Crocker (1991) find that it is possible to have first-best outcomes in a market under adverse selection and moral hazard. Since we are primarily interested in studying the distortions on contracts caused by informational asymmetry, we will restrict our attention to the class of problems in which (3.IC<sub>H</sub>) must bind and thus first-best outcome is not attainable.

#### 2.2.4 Competitive Market with Endogeneity

As discussed at the beginning of this section, the contracts offered will affect  $\bar{p}$  and an individual's risk level under across-contract endogeneity, and hence each individual's utility and profit. Nevertheless, since each insurer has only a small share in the competitive market, she will disregard the externality of the contracts offered on the average level of risk in the market. As a result, each insurer will take  $\bar{p}$  as fixed and offer *MWS* contracts with respect to this  $\bar{p}$ . The *MWS* contracts will pin down  $\bar{p}$  by a fixed point condition.

Formally, *MWS* contracts with endogenous risk are  $(\bar{c}^{H*}, \bar{c}^{L*}, \bar{p})$  such that  $\bar{c}^{H*} =$

---

<sup>3</sup>The problem is first-best if we view the indirect utility function  $U^i$  and profit function  $\pi_i$  involving the endogenous  $p_i$  as “fundamental”. We provide a more detailed discussion of the use of language of “first-best” and “second-best” in the next section.

$(c_{NL}^{H*}, c_L^{H*})$  and  $\bar{c}^{L*} = (c_{NL}^{L*}, c_L^{L*})$  are the *MWS* contracts given  $\bar{p}$ , and  $\lambda p_L^*(c_{NL}^{L*}, c_L^{L*}; \bar{p}) + (1 - \lambda)p_H^*(c_{NL}^{H*}, c_L^{H*}; \bar{p}) = \bar{p}$ , i.e.,

$$(\bar{c}^{H*}, \bar{c}^{L*}) \in \left\{ \begin{array}{ll} \arg \max_{\bar{c}^H, \bar{c}^L} U^L(\bar{c}^L; \bar{p}) & \\ \text{subject to } U^H(\bar{c}^H; \bar{p}) \geq \tilde{U}^H(\bar{p}) & (4.MU_H) \\ U^H(\bar{c}^H; \bar{p}) \geq U^H(\bar{c}^L; \bar{p}) & (4.IC_H) \\ U^L(\bar{c}^L; \bar{p}) \geq U^L(\bar{c}^H; \bar{p}) & (4.IC_L) \\ \lambda \pi_L(\bar{c}^L; \bar{p}) + (1 - \lambda)\pi_H(\bar{c}^H; \bar{p}) = 0 & (4.\text{Zero profit}) \end{array} \right\} \quad (4)$$

and

$$\{\lambda p_L^*(c_{NL}^{L*}, c_L^{L*}; \bar{p}) + (1 - \lambda)p_H^*(c_{NL}^{H*}, c_L^{H*}; \bar{p}) = \bar{p}\}. \quad (4.\text{Consistency})$$

The consistency constraint can be viewed as a fixed point condition that the allocation must satisfy. Note that the indirect utility function, the *H* type's first-best utility and the profit function all depend on  $\bar{p}$  due to across-contract endogeneity. Since every competitive insurer treats  $\bar{p}$  as exogenous,  $\bar{p}$  is not a choice variable as in the monopoly's endogenous problem.

## 2.3 Assumptions

As an individual's risk level  $p$  is endogenous, indifference curves and iso-profit curves are not as analytically clean as in a standard insurance market a la Rothschild and Stiglitz (1976) with pure adverse selection. Additional structure is therefore needed to facilitate the analysis.

Before stating our assumptions, it is useful to consider the one-agent first-best problem under monopoly without across-contract endogeneity:

$$\begin{aligned} \max_{c_{NL}^i, c_L^i} \quad & \pi_i(c_{NL}^i, c_L^i) \\ \text{subject to } & U^i(c_{NL}^i, c_L^i) \geq \bar{U}^i \equiv \bar{U}^i + R, \end{aligned} \quad (5) \quad (IR_i(R))$$

where  $R$  is the utility rent of type  $i$ .<sup>4</sup> It is straightforward to show that  $(IR_i(R))$  must bind in the solution to (5). Thus the problem is equivalent to one that maximizes profits by choosing the profit-maximizing contract on a given indifference curve  $(\bar{U}^i + R)$ . Since indifference curves are downward sloping, we can parametrize the indifference curve associated with a given level of rent  $R$  via  $c_{NL}^i$ .

---

<sup>4</sup>We would like to stress again that the problem is first-best from the point of view of a market with moral hazard.

Denote the profit function restricted on type  $i$ 's indifference curve with rent  $R$  by

$$\pi_i^R(c_{NL}) \equiv \pi_i(c_{NL}, \tilde{c}_L(c_{NL}; R))$$

where  $\tilde{c}_L(c_{NL}; R)$  is the solution to  $U^i(c_{NL}, c_L) = \bar{U}^i + R$

and the correspondence of the first-best contracts with respect to  $R$  by

$$c^i(R) = \left\{ (c_{NL,R}^{i*}, c_{L,R}^{i*}) : (c_{NL,R}^{i*}, c_{L,R}^{i*}) \in \arg \max_{c_{NL}^i, c_L^i} \pi_i(c_{NL}^i, c_L^i) \right. \\ \left. \text{subject to } (IR_i(R)) \right\}.$$

Assuming the solution to the first-best problem (5) is unique for each  $R$ , the correspondence  $c^i(R)$  is single-valued for each  $R$ . By the Theorem of the Maximum, it is a continuous function and we will denote it as  $\vec{c}^i(R)$ .

The dual of the problem (5) can be thought of as maximizing an individual's utility subject to a generalized "budget constraint". As the individual has a higher "budget", it is not unreasonable for her to consume more of both goods, i.e., we expect consumptions in both loss and no loss states to be normal goods. This leads to the following definition.

**Definition 1. (*Well-behavedness*)** *The first-best problem (5) for type  $i$  is **well-behaved** if  $\pi_i^R(c_{NL}^i)$  is single-peaked for any  $R$  and  $\vec{c}^i(R)$  increases monotonically in both  $c_{NL}$  and  $c_L$  with  $R$ .*

The following assumption ensures that the comparative statics of the contracts is well-behaved.

**Assumption 1.** *The first-best problem (5) for the  $L$  and  $H$  type is well-behaved.*

For the moment, we treat Assumption 1 as a "high-level" assumption. But (a) we have verified that it holds, in practice, for a wide range of natural specifications, and (b) it is straightforward to provide sufficient conditions on fundamentals which ensure that it holds.<sup>5</sup>

In addition, we will impose a certain structure on the cost function  $h_i(p, \bar{p})$ . It is useful to first define the compensated elasticity in the following manner.

**Definition 2. (*Compensated elasticity*)** *Let  $\eta_{p,\bar{p}} = \frac{\bar{p}}{p} \frac{\partial p}{\partial \bar{p}} \Big|_h$  be the **compensated elasticity** of  $p$  with respect to  $\bar{p}$  holding  $h(p, \bar{p})$  constant.*

Note that the compensated elasticity is a function of the contract. A particular convenient class of  $h$  functions for analysis is the set of functions  $h(p, \bar{p})$  that solely depends on the ratio between powers of  $p$  and  $\bar{p}$ , i.e.,  $h(p, \bar{p}) = \hat{h}(\frac{p}{\bar{p}})$  for some  $\alpha \neq 0$ .

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<sup>5</sup>See Lemma 5 in the Appendix for a sufficient condition for monotonically increasing  $\vec{c}^i(R)$ .

Since  $\eta_{p,\bar{p}} = \frac{\bar{p}}{p} \frac{\partial p}{\partial \bar{p}} \Big|_h = \frac{\bar{p}}{p} \frac{\frac{\partial \hat{h}}{\partial \bar{p}}}{\frac{\partial \hat{h}}{\partial p}} = \frac{\bar{p}}{p} \frac{-\hat{h}' \frac{p^\alpha}{\bar{p}^2}}{\hat{h}' \frac{\alpha p^{\alpha-1}}{\bar{p}}} = -\frac{1}{\alpha}$ , where  $\hat{h}' = \frac{\partial \hat{h}(\frac{p^\alpha}{\bar{p}})}{\partial \frac{p^\alpha}{\bar{p}}}$ , we know that the elasticity is independent of contract. In fact, this class belongs to a more general class of functions that exhibits a consistent ordering of compensated elasticity between the two types, in particular  $|\eta_{p,\bar{p}}^H| \leq |\eta_{p,\bar{p}}^L|$  at any given point. Our results go through in the more general ordering of compensated elasticity.

### 3 Analysis

#### 3.1 Monopoly Market without Endogeneity

The following proposition characterizes the solution to the monopoly's problem without endogeneity.

**Proposition 1.** *There exists  $\underline{\lambda}$  and  $\bar{\lambda}$  such that*

1. *If  $0 \leq \lambda < \underline{\lambda}$ , the optimal L-type contract is the endowment  $(W, W - D)$ , and the optimal H-type contract is the first-best contract with  $R = 0$ .<sup>6</sup> The solution is unique.*
2. *If  $\underline{\lambda} < \lambda < \bar{\lambda}$ , the H-type contract is first-best with  $R > 0$  and the L-type contract provides positive insurance and zero utility rents.<sup>7</sup>*
3. *If  $\lambda > \bar{\lambda}$ , then  $(c_{NL}^L, c_L^L) = (c_{NL}^H, c_L^H)$  and H types earn positive rents  $R > 0$ . The solution is unique.<sup>8</sup>*

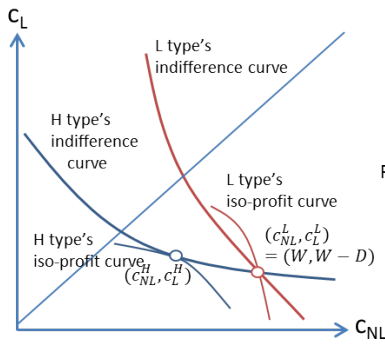


Figure 1:  $0 \leq \lambda < \underline{\lambda}$

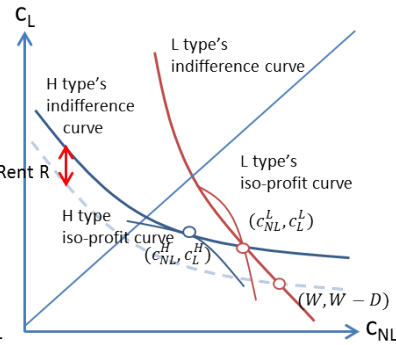


Figure 2:  $\underline{\lambda} < \lambda < \bar{\lambda}$

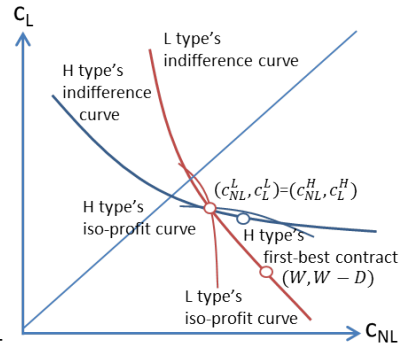


Figure 3:  $\lambda > \bar{\lambda}$

To gain intuition on the solution, we first illustrate the three regimes in Proposition 1 graphically.

<sup>6</sup>Again, the contract is first-best in the sense that we take  $U^i$  and  $\pi_i$ , which are derived from considering the endogeneity of  $p$  due to moral hazard, as fundamentals.

<sup>7</sup>When  $\lambda = \underline{\lambda}$ , the L-type contract being the endowment  $(W, W - D)$  and the optimal H-type contract being the first-best contract with  $R = 0$  are optimal. However, it is possible for contracts of the form stated in (2) to be optimal.

<sup>8</sup>When  $\lambda = \bar{\lambda}$ , the solution can be of the form stated in (2) or (3).

Figure 1 shows the optimal contracts for sufficiently low values of  $\lambda$ . To earn profits on  $L$  types, the monopolist needs to give rents to  $H$  types. Since there are too few  $L$  types in the market, the monopolist prefers to forgo profits on  $L$  types and extract maximal profits from  $H$  types instead. Under such circumstances,  $H$  types will purchase their first-best contract while  $L$  types do not purchase any insurance at all. Figure 2 illustrates the optimal contracts when  $\lambda$  exceeds  $\underline{\lambda}$ . As the fraction of  $L$  types rises, the monopolist eventually finds it desirable to give positive rents to  $H$  types in order to sell profitable contracts to  $L$  types.

As  $\lambda$  rises, the monopolist gives higher and higher rents to  $H$  types, and the optimal  $L$ -type contract moves up and to the left along the  $L$  type's indifference curve through the endowment point. It is possible that, when these rents get sufficiently high, the first-best  $H$ -type contract for the required level of rent lies to the right of this  $L$  type's indifference curve, as depicted in Figure 3. As this would fail to be incentive compatible, pooling the  $H$ -type contract with the  $L$ -type contract maximizes profit. In the limit as  $\lambda \rightarrow 1$ , it becomes optimal to offer  $L$  types their first-best zero-rent contract.

We will prove the proposition in three steps. In the first step, we simplify the set of constraints of the problem (1). In the second step, we introduce a new variable  $R$  to decompose the problem into an outer and inner problem and solve the inner problem. In the third step, we solve the outer problem and fully characterize the solution to the original profit maximization problem. The technical details of the proof are relegated to the Appendix, but we provide the intuition here.

**Step 1:** We show that by simplifying the set of constraints, the problem (1) can be restated as

$$\begin{aligned}
\max_{c_{NL}^H, c_L^H, c_{NL}^L, c_L^L} \quad & \pi = \lambda \pi_L + (1 - \lambda) \pi_H & (6) \\
\text{subject to} \quad & U^L(c_{NL}^L, c_L^L) = \bar{U}^L & (\overline{IR}_L) \\
& U^H(c_{NL}^H, c_L^H) = U^H(c_{NL}^L, c_L^L) & (\overline{IC}_H) \\
& c_{NL}^H \leq c_{NL}^L. & (\text{Monotonicity})
\end{aligned}$$

This equivalence holds for basically standard reasons: the constraint  $(IR_H)$  is redundant; the constraints  $(IR_L)$  and  $(IC_H)$  must bind, and the single crossing property, which is proved in Lemma 6 in the Appendix, implies that  $(IC_L)$  can be replaced with a monotonicity constraint much as in Mirrlees (1971).

**Step 2:** Since  $(IC_H)$  binds in the solution, it is natural to introduce a new variable  $R$ , the utility rent that  $H$  types get. The simplified optimization problem can thus be

decomposed as follows:

$$\begin{aligned}
\max_R \quad & \max_{c_{NL}^H, c_L^H, c_{NL}^L, c_L^L} \lambda \pi_L + (1 - \lambda) \pi_H & (7) \\
\text{subject to } & U^L(c_{NL}^L, c_L^L) = \bar{U}^L & (\overline{IR_L}) \\
& U^H(c_{NL}^H, c_L^H) = U^H(c_{NL}^L, c_L^L) & (\overline{IC_H}) \\
& c_{NL}^H \leq c_{NL}^L & (\text{monotonicity}) \\
& U^H = \bar{U}^H + R. & (\bar{R})
\end{aligned}$$

The inner problem, which we denote as  $ALT(R)$ , pins down the set of profit-maximizing contracts that satisfy  $(\overline{IR_L})$ ,  $(\overline{IC_H})$ , monotonicity and an additional constraint  $(\bar{R})$ .  $(\bar{R})$  specifies the utility of  $H$  types in the solution to be  $\bar{U}^H + R$ . The outer problem is to choose  $R$  to maximize profits. It is clear that the solution to problem (6) is a solution to the combined inner and outer problem (7). Once the problem is decomposed, we can solve the inner problem and study how the contracts offered vary with  $R$ .

**Step 3:** We examine the outer problem and study the comparative statics of  $R$  with respect to  $\lambda$ . We define  $\hat{\pi}_i(R)$  to be the profits from type  $i$  in the solution to  $ALT(R)$  when the rent given to  $H$  types equals  $R$ . The outer problem can be written as

$$\max_R \pi(R, \lambda) = \lambda \hat{\pi}_L(R) + (1 - \lambda) \hat{\pi}_H(R).$$

By showing that the total profit has increasing differences in  $(R, \lambda)$ , we can apply Topkis's Theorem to relate  $R^*$ , the optimal value of rents, with  $\lambda$ , the fraction of  $L$  types in the market. In particular, Topkis's Theorem implies that the set of optimal rents is non-decreasing in  $\lambda$  in the strong set order. Therefore, the well-behaved comparative statics of the optimal contracts with respect to  $R$  can be translated to a well-behaved comparative statics of the optimal contracts with respect to  $\lambda$  as claimed in the proposition.

Figure 4 shows how the optimal rent varies with  $\lambda$  if the solution to the outer problem for each  $\lambda$  is unique and  $\bar{\lambda} \leq 1$ .<sup>9</sup> A low value of  $\lambda$  corresponds to a low value of rents, and thus only  $H$  types will purchase insurance in the solution. A higher  $\lambda$  will result in an  $H$ -type first-best contract with positive rents and an  $L$ -type contract with zero rents. If  $\bar{\lambda} \leq 1$ , a sufficiently high  $\lambda$  will result in a pooling contract.

### 3.2 Monopoly Market with Endogeneity

So far, we have characterized the solution to the monopoly's exogenous problem. Now we introduce across-contract endogeneity and study how it will distort the contracts of

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<sup>9</sup>Although there are nice monotone comparative statics, the profit function  $\pi(R, \lambda)$  is not necessarily differentiable. It is theoretically possible that some values of rents are not optimal for any  $\lambda$  and there will be "gaps" in the graph of the optimal rent correspondence.



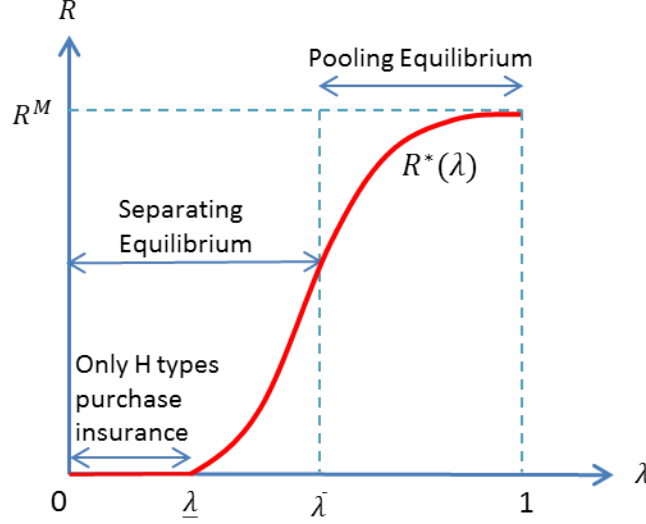


Figure 4: Relationship between  $R^*$  and  $\lambda$

both types. We do so in three steps: (1) simplify the set of constraints, (2) study the sign of the Lagrange multiplier on the consistency constraint and (3) look at how contracts are distorted with respect to the optimal contracts without endogeneity.

**Step 1:** We simplify the problem (2) as

$$\begin{aligned}
 \max_{\bar{p}} \max_{c_{NL}^H, c_L^H, c_{NL}^L, c_L^L} \pi &= \lambda \pi_L(c_{NL}^L, c_L^L; \bar{p}) + (1 - \lambda) \pi_H(c_{NL}^H, c_L^H; \bar{p}) & (8) \\
 \text{subject to } U^L(c_{NL}^L, c_L^L; \bar{p}) &= U^L(W, W - D; \bar{p}) & (\overline{IR}_L) \\
 U^H(c_{NL}^H, c_L^H; \bar{p}) &= U^H(c_{NL}^L, c_L^L; \bar{p}). & (\overline{IC}_H) \\
 c_{NL}^H &\leq c_{NL}^L & (\text{Monotonicity}) \\
 \lambda p_L^*(c_{NL}^L, c_L^L; \bar{p}) + (1 - \lambda) p_H^*(c_{NL}^H, c_L^H; \bar{p}) &= \bar{p}. & (\text{Consistency})
 \end{aligned}$$

In other words,  $(IR_H)$  is redundant,  $(IR_L)$  and  $(IC_H)$  must bind in the solution and  $(IC_L)$  can be replaced by monotonicity. The proof is similar in spirit to the one in the monopoly's exogenous problem. Note that the only difference is that we ought to consider movements of contracts that keep the ambient level of risk constant, e.g., a movement of contract along the iso-probability line.

For the simplified optimization problem, the Lagrangian can be expressed as

$$\begin{aligned}
 \mathcal{L}(c_{NL}^L, c_L^L, c_{NL}^H, c_L^H, \bar{p}) &= \lambda \pi_L(c_{NL}^L, c_L^L; \bar{p}) + (1 - \lambda) \pi_H(c_{NL}^H, c_L^H; \bar{p}) \\
 &+ \alpha (U^L(c_{NL}^L, c_L^L; \bar{p}) - U^L(W, W - D; \bar{p})) \\
 &+ \beta (U^H(c_{NL}^H, c_L^H; \bar{p}) - U^H(c_{NL}^L, c_L^L; \bar{p})) + \gamma (c_{NL}^L - c_{NL}^H) \\
 &+ \mu (\lambda p_L^*(c_{NL}^L, c_L^L; \bar{p}) + (1 - \lambda) p_H^*(c_{NL}^H, c_L^H; \bar{p}) - \bar{p}), & (9)
 \end{aligned}$$

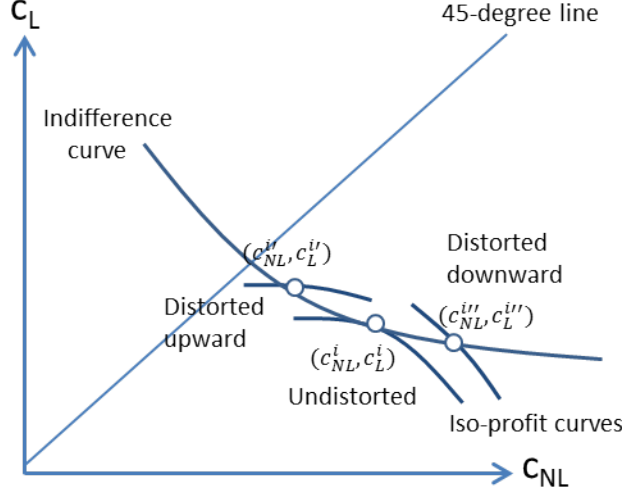


Figure 5: Possible distortions on an undistorted contract

where  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\mu$  are the Lagrange multipliers on  $(\overline{IR}_L)$ ,  $(\overline{IC}_H)$ , the monotonicity constraint and the consistency constraint respectively.

**Step 2:** Now that we have defined the Lagrangian for the problem, it will be useful to determine the sign of  $\mu$ , which has implications on the optimal average level of risk in the market. A positive sign signifies that in the absence of the consistency constraint, the monopolist would have chosen contracts that yield a lower  $\bar{p}$  to improve her profits, and vice versa. The sign also tells us the direction of distortion on each type's contract under across-contract endogeneity.

We first look at the one-agent profit maximization problem to gain insight into the sign of the multiplier. When only one type is present, the incentive compatibility and monotonicity constraints are trivial, so the optimization problem can be simplified as

$$\begin{aligned} \max_{\bar{p}} \max_{c_{NL}, c_L} \pi(c_{NL}, c_L; \bar{p}) & \quad (10) \\ \text{subject to } U(c_{NL}, c_L; \bar{p}) &= U(W, W - D; \bar{p}) & (\overline{IR}) \\ p^*(c_{NL}, c_L; \bar{p}) &= \bar{p} & (\text{Consistency}) \end{aligned}$$

and the corresponding Lagrangian is

$$\begin{aligned} \mathcal{L}(c_{NL}, c_L, \bar{p}) &= \pi(c_{NL}, c_L; \bar{p}) + \alpha (U(c_{NL}, c_L; \bar{p}) - U(W, W - D; \bar{p})) \\ &\quad + \mu (p^*(c_{NL}, c_L; \bar{p}) - \bar{p}). \end{aligned}$$

We define the following terms to describe possible distortions, which are easily visualized as in Figure 5.

**Definition 3. (*Distortions*)** A contract  $(c_{NL}, c_L)$  is **undistorted** if the indifference

curve is tangent to the iso-profit curve at  $(c_{NL}, c_L)$ . It is **distorted upward** if the indifference curve is steeper than the iso-profit curve at  $(c_{NL}, c_L)$ . It is **distorted downward** if the indifference curve is less steep than the iso-profit curve at  $(c_{NL}, c_L)$ .

The following lemma summarizes the direction of distortion by looking at the implication of the sign of  $\mu$  on profits for a small local movement of the contract.

**Lemma 1.** *In a one-type problem, the optimal contract is distorted upward if  $\mu$  is positive and distorted downward if  $\mu$  is negative.*

*Proof.* See Appendix Section C. □

When  $\mu$  is positive, a small movement of the contract along the indifference curve towards the endowment point holding  $\bar{p}$  constant will increase profits. This implies that the contract offered involves more insurance relative to the undistorted contract. On the contrary, when  $\mu$  is negative, a small movement of the contract along the indifference curve away from the endowment point holding  $\bar{p}$  constant will increase profits. This implies that the contract offered involves less insurance relative to the undistorted contract.

It is not hard to find an example in which  $\mu$  can take either a positive or a negative sign. We consider a one-type case and choose the following functional forms and parameters:  $u(x) = -\frac{1}{2x^2}$ ,  $h(p, \bar{p}) = \frac{1}{100(1-\bar{p})^3(1+p)^3}$ ,  $W = 4$  and  $D = 2$ .<sup>10</sup>

Figure 6 shows the indifference curves that pass through the endowment when  $\bar{p}_1 = 0.3$  (the pink line) and  $\bar{p}_2 = 0.6$  (the blue line). On each of the indifference curves, the red square marker indicates the contract  $(c_{NL}, c_L)$  where the consistency constraint binds, i.e.,  $p^*(c_{NL}, c_L; \bar{p}) = \bar{p}$ , while the black circle marker indicates the contract that maximizes the profits for a fixed  $\bar{p}$ , i.e., maximizes profit while only satisfying the individual rationality constraint but not necessarily the consistency constraint.

On the indifference curve with  $\bar{p}_1 = 0.3$ , the profit-maximizing contract lies to the right of the consistency-constraint contract – moving the contract from the consistency-constraint contract along the indifference curve to the right holding  $\bar{p}_1$  constant will increase profits. Thus the optimal contract is distorted upward. By Lemma 1,  $\mu$  is positive. On the contrary, when  $\bar{p}_2 = 0.6$ , the profit-maximizing contract lies to the left of the consistency-constraint contract – moving the contract from the consistency-constraint contract along the indifference curve to the left holding  $\bar{p}_2$  constant will increase profits. Thus the optimal contract is distorted downward. By Lemma 1,  $\mu$  is negative.

The above example demonstrates that it is possible for  $\mu$  to be positive or negative in a one-agent profit maximization problem. Therefore, in step 3, when we analyze how contracts are distorted by the across-contract endogeneity, we will consider both cases: (1)  $\mu$  is positive and (2)  $\mu$  is negative.

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<sup>10</sup>This particular functional form for  $h$  may yield negative  $p$  or large  $p > 1$  when we solve the first-order condition for utility maximization. It is possible to patch the  $h$  function for extreme values of  $p$  in a way that has no effect on the equilibrium.

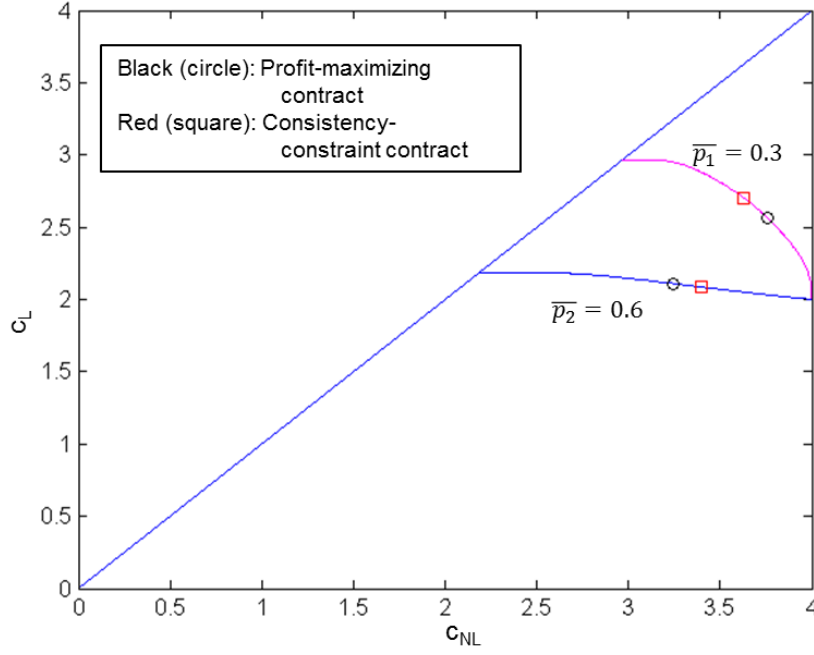


Figure 6: An one-agent example

**Step 3:** To study how each type’s contract is distorted, we divide our analysis into two parts: separating contracts and pooling contracts.

It is worth noting that both moral hazard and across-contract endogeneity are present in the model. The undistorted contract per Definition 3 is first-best if we view the indirect utility function and the profit function derived from taking the endogenous  $p$  response into account as “fundamental”. However, from the viewpoint of a market with pure adverse selection, the undistorted contract is second-best, because individuals only purchase insurance with partial coverage due to moral hazard.<sup>11</sup> Across-contract endogeneity further distorts the second-best contract so an optimal contract in a market with interdependence of risks can be viewed as third-best. To state the distortions we are examining precisely and avoid the confusing language of first-, second- and third-best, we give the following definitions.

**Definition 4. (*Distortions on a pooling contract*)** Consider a pooling contract  $(c_{NL}, c_L)$ . Then the pooling contract is ***p-distorted upward*** if the  $L$  type’s indifference curve is steeper than the pooled iso-profit curve at  $(c_{NL}, c_L)$ ; it is ***p-distorted downward*** if the  $L$  type’s indifference curve is less steep than the pooled iso-profit curve at  $(c_{NL}, c_L)$ .

Figure 7 illustrates the distortions on a pooling contract. The distortions on a pooling contract are similar to those on an undistorted contract in the sense that both are defined in terms of the relative steepness of the slopes of an indifference curve and an iso-profit curve. However, we consider the pooled iso-profit curve, instead of a particular type’s

<sup>11</sup>The undistorted contract is also not first-best with endogenous contracts because of the externality.

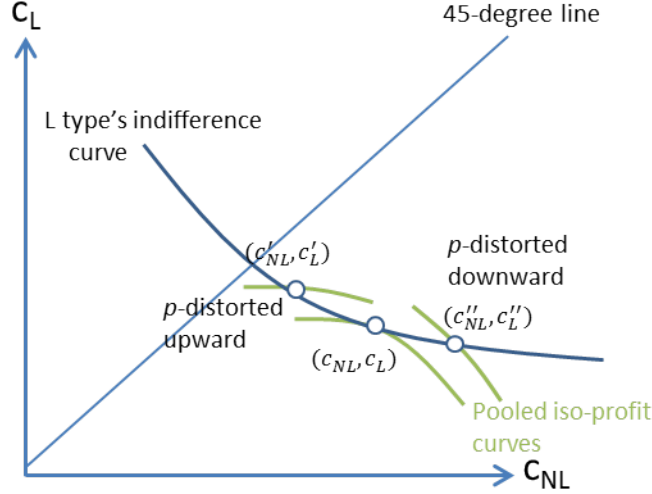


Figure 7: Possible distortions on a pooling contract

iso-profit curve, for distortions on a pooling contract, since such distortions will influence the profits from both types.

For separating contracts,  $H$  types are informationally unconstrained so the distortions on the  $H$ -type contract are analogous to the ones on an undistorted contract. Yet, it is trickier to define distortions on the  $L$  type's contract due to the binding of  $(IC_H)$ . A movement of the  $L$  type's contract must be accompanied by an incentive compatible movement of the  $H$  type's contract. Therefore, distortions on the  $L$  type's contract are defined by joint movements of both type's contracts.

**Definition 5. (*Distortions on separating contracts*)** Consider separating contracts  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  that satisfy  $(\overline{IC}_H)$ . The  $H$ -type contract is ***s-distorted upward*** if the  $H$ -type contract is distorted upward; it is ***s-distorted downward*** if the  $H$ -type contract is distorted downward. The  $L$ -type contract is ***s-distorted upward*** if moving the  $H$ -type contract down along the iso-probability line to the left and  $L$ -type contract down along the  $L$  type's indifference curve to the right, fixing  $\bar{p}$  and maintaining  $(\overline{IC}_H)$ , increases profits; it is ***s-distorted downward*** if moving the  $H$ -type contract up along the iso-probability line to the right and  $L$ -type contract up along the  $L$  type's indifference curve to the left, fixing  $\bar{p}$  and maintaining  $(\overline{IC}_H)$ , increases profits.

An  $s$ -upward distorted and an  $s$ -downward distorted  $L$ -type contract are depicted in Figure 8 and Figure 9 respectively.

Intuitively, if  $\bar{p}$  is the optimal level of average risk in the endogenous problem, the solution to the exogenous problem associated with  $\bar{p}$  will not be a solution to the endogenous problem – the optimal endogenous contracts are distorted because of the addition of the consistency constraint. The sign of  $\mu$ , which can be viewed as the “penalty” for violating the consistency constraint, suggests the directions in which the monopoly insurer will

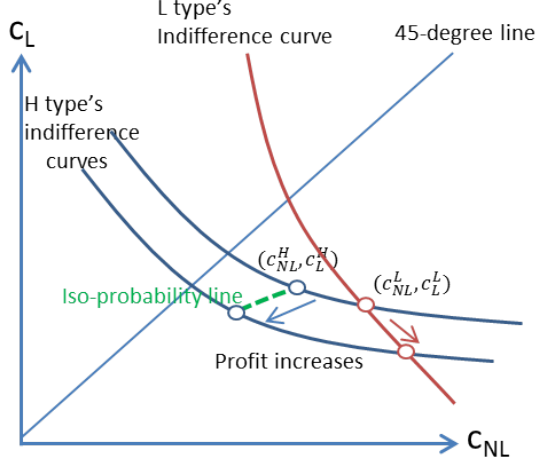


Figure 8: An  $s$ -upward distorted  $L$ -type contract

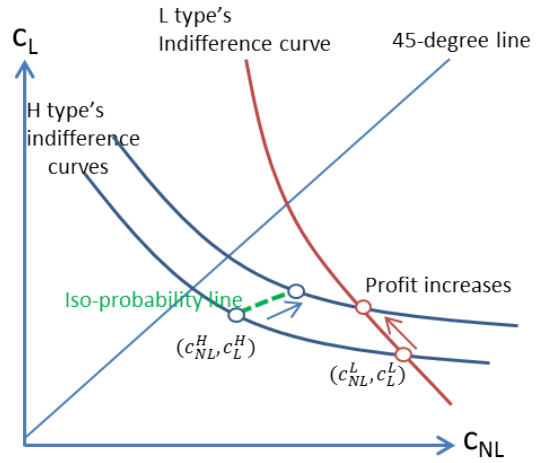


Figure 9: An  $s$ -downward distorted  $L$ -type contract

distort the contracts. By considering the directional derivative of the Lagrangian in particular directions, we can determine what those distortions are. The following proposition specifies the distortions on the optimal contracts.

**Proposition 2.** *Suppose  $\mu$  is positive. If the optimal contracts are identical (i.e., pooling contract), then the pooling contract is  $p$ -distorted upward; if the optimal contracts are distinct (i.e., separating contract), then both  $H$  type's and  $L$  type's contracts are  $s$ -distorted upward.*

*Suppose  $\mu$  is negative. If the optimal contracts are identical (i.e., pooling contract), then the pooling contract is  $p$ -distorted downward; if the optimal contracts are distinct (i.e., separating contract), then both  $H$  type's and  $L$  type's contracts are  $s$ -distorted downward.*

*Proof.* See Appendix Section D. □

Proposition 2 establishes the directions of distortions on the optimal contracts with endogeneity. As we have discussed, a positive  $\mu$  encourages the monopolist to choose a higher level of average risk in the market – individuals are likely to purchase more insurance when the market is riskier and hence the monopolist can profit from selling more insurance. A negative  $\mu$  leads to a lower level of average risk in the market – in a less risky environment, both types are better off and the monopolist can extract more profits from the individuals. In principle, both cases are possible, but we argue that the negative case may be more relevant, as lower risk is often deemed better and it is less likely that insurers would attempt to raise profits by deliberately creating a riskier environment.

### 3.3 Competitive Market without Endogeneity

As in the monopoly's case, (3. $IC_L$ ) can be replaced by the monotonicity constraint. The arguments are essentially identical so we omit them here.

To characterize what the optimal contracts in a competitive market without across-contract endogeneity are, we first pin down the set of possible candidates. We note that because (3. $IC_H$ ) binds in the solution, a natural way to proceed is to parametrize the set of candidate allocations by values of  $U^H$ . For a given  $\lambda$ , define the pooled zero-profit curve as the set of contracts  $(c_{NL}, c_L)$  such that pooling the contract yields zero profit, i.e.,  $\lambda\pi_L(c_{NL}, c_L) + (1 - \lambda)\pi_H(c_{NL}, c_L) = 0$ .

For a given  $U^H \geq \tilde{U}^H$  that is feasible, one possible candidate is the pooling zero-profit contract, which is located at the intersection between the pooled zero-profit curve and the  $H$  type's indifference curve  $U^H$ . It is clear that a pooling zero-profit contract satisfies (3. $MU_H$ ), (3. $IC_H$ ) and monotonicity, and we will characterize the condition under which the pooling zero-profit contract maximizes the  $L$  type's utility for feasible values of  $U^H$ .<sup>12</sup>

**Lemma 2.** *For a feasible level of  $U^H$ , the associated pooling zero-profit contract will be the  $U^L$ -maximizing allocation if and only if moving the  $H$ -type contract from the pooling zero-profit contract to the left along the  $H$  type's indifference curve does not yield higher profits from  $H$  types.*

*Proof.* See Appendix Section E. □

The idea behind the lemma is straightforward. There are essentially two cases: (1) the  $H$  type's profit-maximizing contract lies on the left of the pooling zero-profit contract and (2) the  $H$  type's profit-maximizing contract lies on or to the right of the pooling zero-profit contract, illustrated by Figure 10 and Figure 11 respectively. Case (1) implies that we can find another contract on the  $H$  type's indifference curve that yields higher profits from  $H$  types than the pooling zero-profit contract without violating the monotonicity constraint, and therefore the corresponding  $U^L$ -maximizing contract is one on the same  $H$  type's indifference curve that yields a profit of  $-\frac{(1-\lambda)\pi_H}{\lambda}$  (for  $\lambda \neq 0$ ). No such  $H$  type's contract can be found in case (2), and thus the pooling zero-profit contract maximizes  $U^L$ .

So far we have looked at the candidate allocation for each feasible  $U^H$ , and it is useful to group these allocations together for characterizing the  $MWS$  contracts. Define the  $L$ -type contract locus to be the set of  $U^L$ -maximizing  $L$ -type contracts for values of  $U^H$  where moving the  $H$ -type contract to the left of the pooling zero-profit contract along the  $H$  type's indifference curve raises profit. Moreover, define the  $L$ -type candidate allocation

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<sup>12</sup>Feasibility of  $U^H$  has a geometrical interpretation – for an infeasible value of  $U^H$ , the associated  $H$  type's indifference curve will not intersect the pooled zero-profit curve. With the assumption of single-peakedness of profit along an indifference curve and the single crossing property, there are no feasible allocations when all the contracts on the  $H$  type's indifference curve, if pooled, yield negative profits.

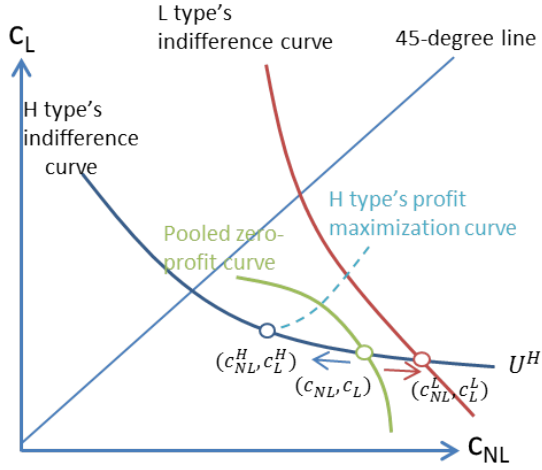


Figure 10: Pooled zero-profit contract does not maximize  $U^L$

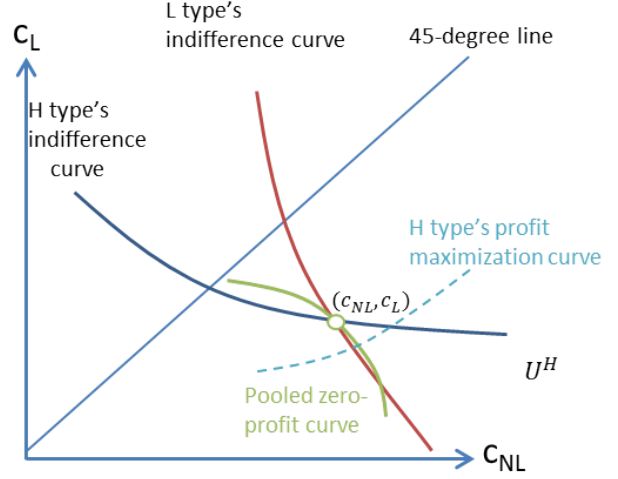


Figure 11: Pooling zero-profit contract maximizes  $U^L$

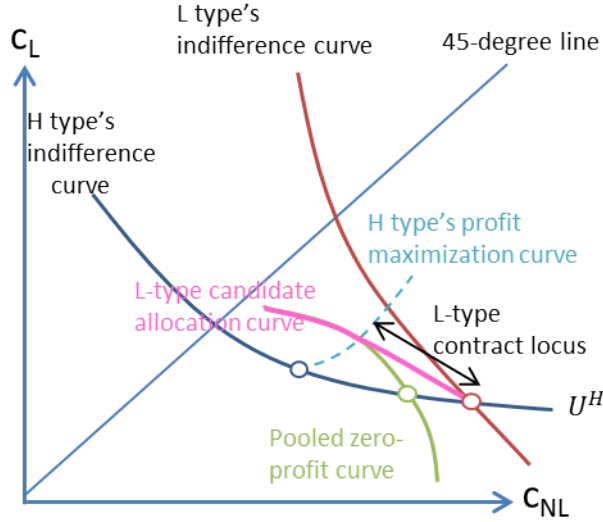


Figure 12: The  $L$ -type candidate allocation curve

curve to be the union of the  $L$ -type contract locus and the portion of the pooled zero-profit curve above its intersection with the  $H$ -type profit maximization curve (i.e., the part of pooled zero-profit curve associated with  $U^H$  where the pooling zero-profit contract maximizes  $U^L$ ). Figure 12 illustrates the  $L$ -type contract locus and the  $L$ -type candidate allocation curve.

As we have identified the candidates for the *MWS*  $L$ -type contract, we are ready to characterize the solution per the following lemma.

**Lemma 3.** *Let  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  be the optimal *MWS* contracts for  $H$  types and  $L$  types respectively. Let  $U^H = U^H(c_{NL}^L, c_L^L)$ . The *MWS* equilibrium  $L$ -type contract is the  $U^L$ -maximizing contract on the  $L$ -type candidate allocation curve while the  $H$ -type contract is the profit-maximizing contract associated with  $U^H$  subject to monotonicity.*



*Proof.* Let  $(c_{NL}, c_L)$  be the pooling zero-profit contract on the indifference curve  $U^H$ . If profits from  $H$  types increase from  $(c_{NL}, c_L)$  to the left, then the  $U^L$ -maximizing contract  $(c_{NL}^L, c_L^L)$  lies on the  $L$ -type contract locus as defined above. If profits from  $H$  types do not increase from  $(c_{NL}, c_L)$  to the left, then the  $U^L$ -maximizing contract contract  $(c_{NL}^L, c_L^L)$  is simply the pooling zero-profit contract  $(c_{NL}, c_L)$ . Under both cases, the  $L$ -type  $MWS$  contract lies on the  $L$ -type candidate allocation curve. The optimality of the contract implies that it must be the one that maximizes  $U^L$  among all the contracts on the  $L$ -type candidate allocation curve. The location of the  $H$ -type contract follows from the proof of Lemma 2 and it maximizes profits from  $H$  types subject to monotonicity.  $\square$

We provide two examples below to demonstrate that the optimal  $MWS$  contracts can be separating or pooling. We use the following functional forms:  $u(x) = -\frac{1}{2x^2}$  and  $h(p, \bar{p}) = \frac{k}{100(1-\bar{p})^3(1+p)^3}$ , with  $\bar{p} = 0.3$ .

In the first example, we choose the following set of parameters:  $\lambda = 0.7$ ,  $W = 4$ ,  $D = 2$ ,  $k_L = 1$  and  $k_H = 3$ . A separating equilibrium is observed and illustrated in Figure 13. The  $H$ -type and  $L$ -type contracts are distinct – the former lies on the  $H$ -type profit maximization curve while the latter lies on the  $L$ -type contract locus.

In the second example we choose the following set of parameters:  $\lambda = 0.9$ ,  $W = 5$ ,  $D = 3$ ,  $k_L = 1$  and  $k_H = 4$ . A pooling equilibrium is observed and illustrated in Figure 14. Notice that the profit maximization point, marked by the intersection of the light blue  $H$ -type profit maximization curve and the dark blue  $H$ -type indifference curve, lies on the right of the pooling zero-profit contract.

### 3.4 Competitive Market with Endogeneity

Competitive insurers do not internalize the externality of insurance contracts – the small market share prompts each insurer to take the average market risk as fixed instead of endogenous. Thus the contracts offered are the optimal contracts in the exogenous problem associated with a certain  $\bar{p}$  and satisfy the consistency constraint as a fixed point condition. Therefore, we would expect the competitive insurers to choose contracts that induce an average risk level  $\bar{p}$  higher than necessary.

The expectation is actually true. We show that the  $MWS$  contracts with endogeneity are inefficient in two steps. First, we consider a family of problems that differ in the exogenous levels of  $\bar{p}$  and show that a parametric lowering of  $\bar{p}$  implies that the constrained Pareto frontier is strictly further out. Second, we show that the  $MWS$  endogenous allocation is constrained Pareto inefficient. At an  $MWS$  endogenous allocation, moving the  $H$ -type contract in an incentive compatible way has no effects from the viewpoint of the insurers because such a movement does not affect profit. However, a notional social planner will recognize the positive welfare effect of the movement via the consistency constraint. A lower  $\bar{p}$  shifts the Pareto frontier out by step 1 and both types will be

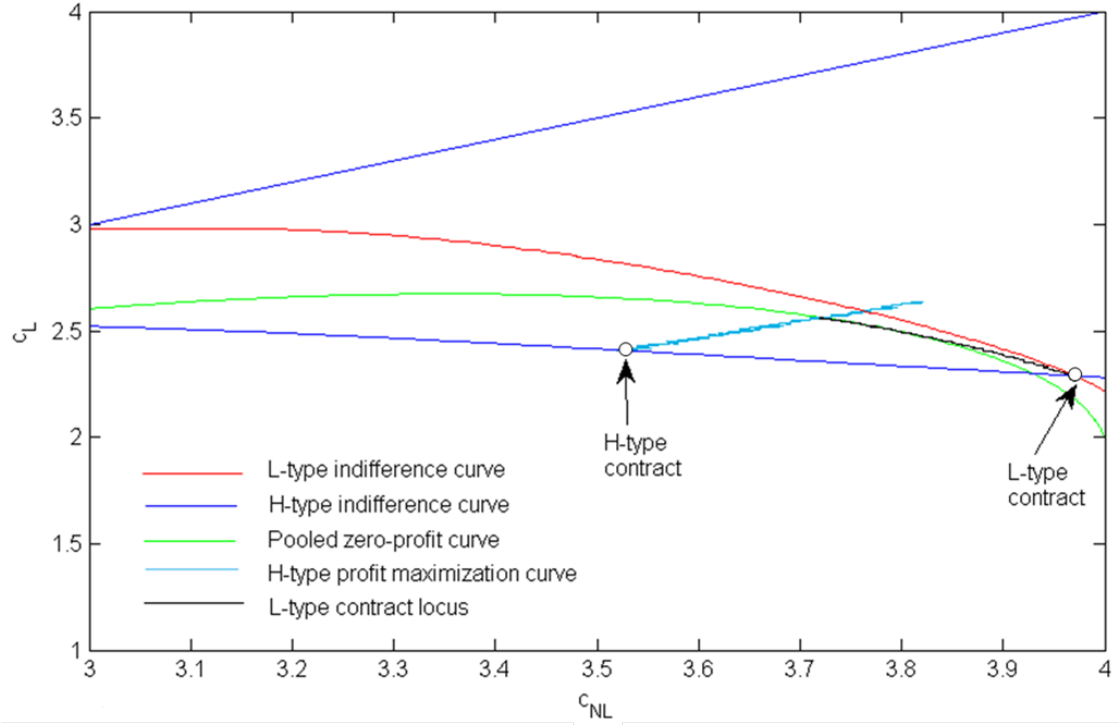


Figure 13: A numerical example of separating contracts

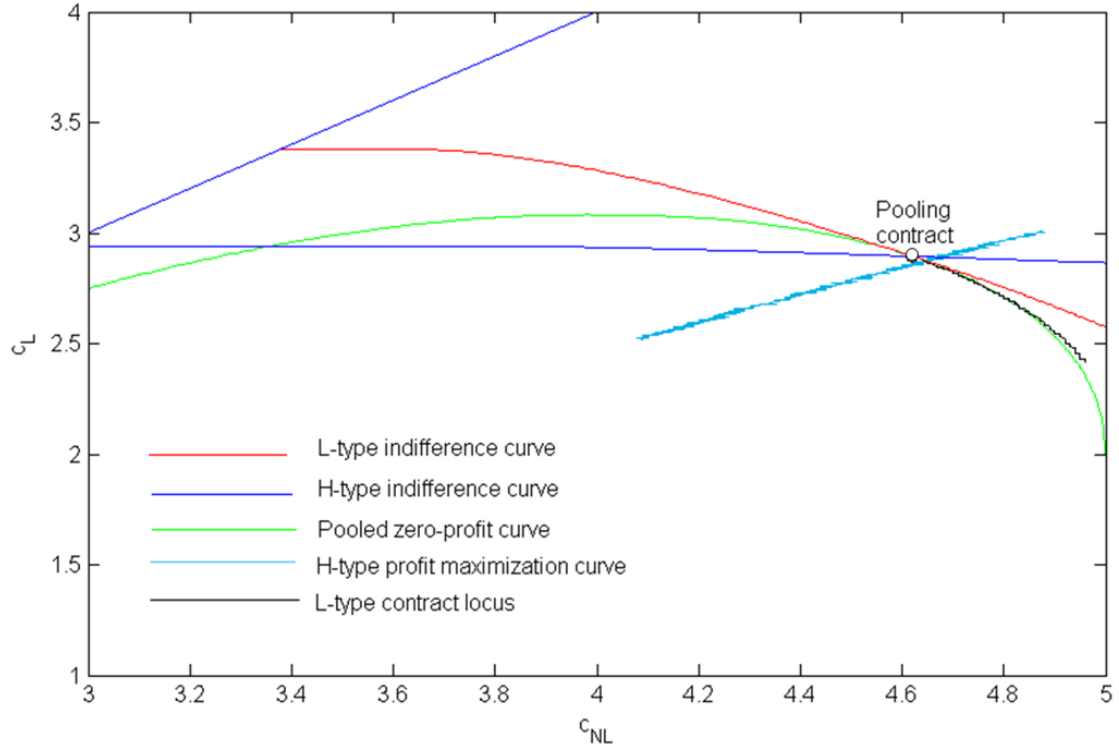


Figure 14: A numerical example of pooling contract

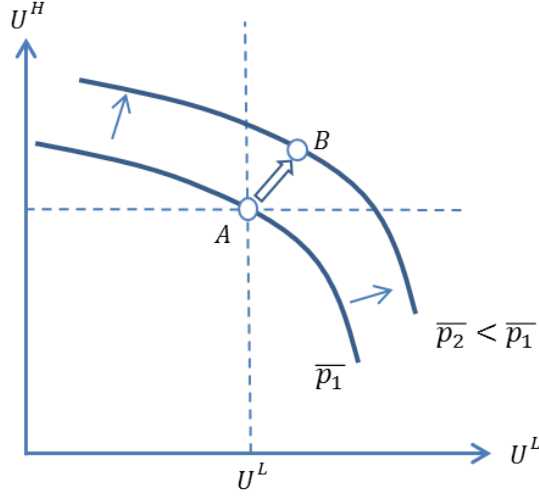


Figure 15: The movement of the Constrained Pareto frontier when  $\bar{p}$  decreases

better off.

In the first step, it is helpful to consider the following program:

$$\begin{aligned}
 F(\bar{p}, \tilde{U}^H) &\equiv \max_{\bar{c}^H, \bar{c}^L} U^L(\bar{c}^L; \bar{p}) \\
 \text{subject to } &U^H(\bar{c}^H; \bar{p}) \geq \tilde{U}^H \quad (MU'_H) \\
 &(3.IC_H), (Monotonicity) \text{ and } (3.Zero \text{ profit}).
 \end{aligned}$$

The program  $F$  and the  $MWS$  exogenous program differ only in the participation constraint of  $H$  types – the outside option is fixed in  $(MU'_H)$  while it is dependent on  $\bar{p}$  in  $(3.MU_H)$ . Indeed the  $MWS$  exogenous program is a special case of the program  $F$ . Note that the program  $F$  does not consider the consistency constraint.

The following lemma establishes that if we can reduce  $\bar{p}$  at any allocation parametrically, then both types will be better off. The result relies on the assumption about the global structure of the  $h$  function, that the difference between  $\eta_{p,\bar{p}}$  at a point between the two types is weakly monotonic, and in particular  $|\eta_{p,\bar{p}}^H| \leq |\eta_{p,\bar{p}}^L|$  at a point.

**Lemma 4.** *The constrained Pareto frontier of the program  $F$  is ordered by  $\bar{p}$ ; in particular, any constrained optimal allocation associated with  $\bar{p}$  is dominated by some feasible outcomes with  $\bar{p}'$  when  $\bar{p}' < \bar{p}$ .*

*Proof.* See Appendix Section F. □

In other words, the Pareto frontier will be “pushed out”, as depicted in Figure 15, if we can reduce  $\bar{p}$  parametrically. Of course, we cannot freely do so because of the consistency constraint. If we can ignore the consistency constraint though, Lemma 4 implies that we will be able to get a Pareto improvement. We will show that we can ignore the consistency constraint at the margin starting from an  $MWS$  endogenous allocation.

**Definition 6. (Constrained Pareto efficiency)** An allocation  $(\bar{c}^H, \bar{c}^L, \bar{p})$  satisfies constrained Pareto efficiency in a competitive market with across-contract endogeneity if it solves

$$\begin{aligned} & \max_{\bar{p}} \max_{\bar{c}^H, \bar{c}^L} U^L(\bar{c}^L; \bar{p}) & (11) \\ & \text{subject to } U^H(\bar{c}^H; \bar{p}) \geq \tilde{U}^H & (MU'_H) \\ & (4.IC_H), (Monotonicity), (4.Zero\ profit) \text{ and } (4.Consistency). \end{aligned}$$

Define  $G$  to be the inner program, i.e.,

$$\begin{aligned} G(\bar{p}, \tilde{U}^H) & \equiv \max_{\bar{c}^H, \bar{c}^L} U^L(\bar{c}^L; \bar{p}) \\ & \text{subject to } U^H(\bar{c}^H; \bar{p}) \geq \tilde{U}^H & (MU'_H) \\ & (3.IC_H), (Monotonicity), (3.zero\ profit) \text{ and } (4.Consistency). \end{aligned}$$

A necessary condition for constrained Pareto efficiency at  $\bar{p}^*$  is that  $G(\bar{p}^*, \tilde{U}^H)$  reaches a local maximum at  $\bar{p} = \bar{p}^*$ .

We adopt the techniques from Rothschild and Scheuer (2013) and Chen and Rothschild (2015) in this step. Notice that the program  $G$  is the inner problem of (11) while the outer problem is to choose  $\bar{p}$  to maximize  $U^L$ . The presence of the consistency constraint in the program  $G$  implies that we cannot parametrically reduce  $\bar{p}$  – it is not only a parameter because of the extra constraint.

If we can show that the multiplier on the consistency constraint is zero at an  $MWS$  endogenous allocation, then the program  $G$  coincides with  $F$  locally. By the previous lemma, the value of  $F$  decreases with  $\bar{p}$ , implying that the value of  $G$  also decreases with  $\bar{p}$ . Therefore, the  $MWS$  endogenous allocation cannot solve the problem (11). We note that the program  $G$  differs from  $F$  by the consistency constraint, so the value of  $G$  should be no greater than that of  $F$  given any pair of  $(\bar{p}, \tilde{U}^H)$ . By definition, the endogenous  $MWS$  allocation associated with  $\bar{p}_{MWS}$  satisfies the consistency constraint and is a solution to the program  $F$  with  $(\bar{p}_{MWS}, \tilde{U}^H(\bar{p}_{MWS}))$ . Then we must have  $F(\bar{p}_{MWS}, \tilde{U}^H(\bar{p}_{MWS})) = G(\bar{p}_{MWS}, \tilde{U}^H(\bar{p}_{MWS}))$ . From the point of view of  $F$ , adding in a consistency constraint has no effect, so the multiplier should equal zero. The proposition below formalizes the intuition.

**Proposition 3.** *The  $MWS$  endogenous allocation is not constrained Pareto efficient in a competitive market with endogeneity – there exists a feasible allocation that Pareto dominates the  $MWS$  endogenous allocation.*

*Proof.* See Appendix Section G. □

We have demonstrated that the *MWS* endogenous allocations in a competitive market with endogeneity is inefficient precisely because there exists a feasible allocation that makes both types better off than the *MWS* endogenous allocation.

## 4 Discussion and Conclusion

In this paper, we have studied across-contract endogeneity in two classes of insurance markets: (1) markets with monopoly insurers and (2) markets with perfectly competitive firms. In each case, we first characterize the optimal contracts in the market without endogeneity and examine how the across-contract endogeneity distorts the optimal contracts. Our analysis suggests that the average level of risk in a monopoly market tends to be lower because the monopolist takes across-contract endogeneity into account and adjusts the level of insurance accordingly. Both types of individuals purchase less insurance in the equilibrium and the market is less risky as a result. This is the market-wide analog of standard arguments that moral hazard reduces optimal insurance level; the difference is that it is the across-contract externalities that is further reducing the optimal level of insurance. In contrast, perfectly competitive insurers consider neither the externality exerted by the contracts nor the benefits of offering less coverage at the margin; the resulting level of average risk in the market is too high and the competitive equilibrium is constrained Pareto inefficient.

Our results suggest that there is a trade-off between the monopoly and competitive market. This is related to a result in Mahoney and Weyl (2014), who argue that there can be a trade-off between competition and externality reduction in selection markets. Their mechanism is different from ours, but has a similar flavor – in our model, the inefficiency of perfect competition stems from the failure of insurers to account for the behavioral externality of the contracts. A monopolist generally extracts rents from the agents due to her market power. However, our analysis establishes that in an insurance market with interdependent risks, the monopolist takes the across-contract endogeneity into consideration and therefore provides less insurance that results in a lower average level of risk. In contrast, competitive insurers are price takers and make zero profit, but they do not internalize the externality of contracts in an insurance market with across-contract endogeneity and offer excessive insurance. The average market risk is too high as a consequence.

Since there is a trade-off between competition and efficiency, an important direction for future work is to characterize the precise conditions under which monopoly is preferred to perfect competition in an insurance market with interdependent risks.<sup>13</sup>

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<sup>13</sup>It is easy to come up with a one-type example in which monopoly is preferable. We construct an example using the following functional forms and parameters:  $W = 5$ ,  $D = 3$ , and  $h(p, \bar{p}) = \frac{0.025}{(1-0.655\bar{p})^3(1+p)^3}$  and  $u(x) = -\frac{1}{2x^2}$ .

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# Appendix

## A Proof of a Sufficient Condition for Assumption 1

The following lemma shows a sufficient condition for the  $H$ -type first-best-contract locus to be upward-sloping in the region of interest.

**Lemma 5.** *Let  $(W, W - D)$  be the endowment such that for a given  $H$  type, there exists  $\epsilon > 0$  such that  $c_{NL,R}^{H*} \leq c_{NL,R'}^{H*}$  and  $c_{L,R}^{H*} \leq c_{L,R'}^{H*}$  for all  $0 \leq R < R' < \epsilon$ . Then there exists  $k_L < k_H$  such that  $\vec{c}^{\vec{H}}(R)$  only intersects the  $L$  type's indifference curve that passes through  $(W, W - D)$  once and  $\vec{c}^{\vec{H}}(R)$  monotonically increases in  $c_{NL}$  and  $c_L$  with  $R$  below the  $L$  type's indifference curve.*

*Proof.* Choose  $k_L < k_H$ . If  $\vec{c}^{\vec{H}}(R)$  only intersects the  $L$  type's indifference curve that passes through  $(W, W - D)$  once and  $\vec{c}^{\vec{H}}(R)$  monotonically increases in  $c_{NL}$  and  $c_L$  with  $R$  below the  $L$  type's indifference curve, then we get the required  $k_L$ .

Suppose that  $\vec{c}^{\vec{H}}(R)$  crosses the  $L$  type's indifference curve associated with  $\bar{U}^L$  multiple times. Let  $(\bar{c}_{NL}, \bar{c}_L)$  be the point at which the  $L$  type's indifference curve intersects the 45-degree line and  $\hat{U}^H = U^H(\bar{c}_{NL}, \bar{c}_L)$ . Parametrize the set of intersections between  $\vec{c}^{\vec{H}}(R)$  and the  $H$  type's indifference curve by  $R$ .

Let  $A = \{R \in [0, \hat{U}^H - \bar{U}^H] : \text{there exists } \epsilon > 0 \text{ such that } c_{NL,R'}^{H*} \geq c_{NL,R}^{H*} \text{ for any } R' \in (R - \epsilon, R) \text{ or } c_{NL,R'}^{H*} \leq c_{NL,R}^{H*} \text{ for any } R' \in (R, R + \epsilon)\}$ , i.e.,  $A$  is the collection of rents associated with the downward sloping part of  $\vec{c}^{\vec{H}}(R)$ . Note that the slope of downward sloping  $\vec{c}^{\vec{H}}(R)$  has to be steeper than that of the  $H$  type's indifference curve at every intersection and thus the difference between their slopes is always positive for every intersection associated with rents in  $A$ . Note that since the slope of the  $H$  type's indifference curve is continuous and that of  $\vec{c}^{\vec{H}}(R)$  is semi-continuous, the difference is semi-continuous.  $A$  is compact and the minimum difference  $m > 0$  is attained at some value of  $R$ .

We assume that the slope of the indifference curve is differentiable in  $k_i$  and we know that the slope is uniformly continuous in the region of concern  $\{(c_{NL}, c_L) : c_{NL} \leq W \text{ and } c_L \geq W - D\}$ . Thus there exists  $\delta > 0$  such that if  $k_L \in (k_H - \delta, k_H + \delta)$ , the difference in slopes of the  $H$  type's and  $L$  type's indifference curves at every point will be smaller than  $m$ . This guarantees that  $\vec{c}^{\vec{H}}(R)$  only crosses the  $L$  type's indifference curve once.

□

## B Proof of Proposition 1

We provide the complete proof of Proposition 1 here.

**Step 1:** We show that the constraint set can be simplified in the following way: (i)  $(IR_H)$  is redundant, (ii)  $(IR_L)$  must bind, (iii)  $(IC_H)$  must bind and (iv)  $(IC_L)$  can be

replaced with a monotonicity constraint  $c_{NL}^H \leq c_{NL}^L$ . In other words we prove that the problem (1) is equivalent to

$$\begin{aligned} \max_{c_{NL}^H, c_L^H, c_{NL}^L, c_L^L} \quad & \pi = \lambda \pi_L + (1 - \lambda) \pi_H & (6) \\ \text{subject to } & U^L(c_{NL}^L, c_L^L) = \bar{U}^L & (\overline{IR}_L) \\ & U^H(c_{NL}^H, c_L^H) = U^H(c_{NL}^L, c_L^L) & (\overline{IC}_H) \\ & c_{NL}^H \leq c_{NL}^L. & (\text{Monotonicity}) \end{aligned}$$

Towards simplifying the constraint set, we first prove the following lemmas about the properties of the indirect utility function and the profit function.

**Lemma 6.** *The  $L$  type's and  $H$  type's indirect utility functions satisfy the single crossing property.*

*Proof.* Taking the partial derivatives of  $U^i$  with respect to  $c_{NL}$  and  $c_L$ , we have

$$\frac{\partial U^i}{\partial c_L} = p_i u'(c_L) + \frac{\partial p_i}{\partial c_L} \left( u(c_L) - u(c_{NL}) - \frac{\partial h_i}{\partial p_i} \right)$$

and

$$\frac{\partial U^i}{\partial c_{NL}} = (1 - p_i) u'(c_{NL}) + \frac{\partial p_i}{\partial c_{NL}} \left( u(c_L) - u(c_{NL}) - \frac{\partial h_i}{\partial p_i} \right).$$

By the envelop theorem,  $\frac{\partial p_i}{\partial c_{NL}} = 0$  and  $\frac{\partial p_i}{\partial c_L} = 0$ . Hence the last term in each expression equals 0, and so

$$\frac{\frac{\partial U^i}{\partial c_{NL}}}{\frac{\partial U^i}{\partial c_L}} = \frac{1 - p_i}{p_i} \left( \frac{u'(c_{NL})}{u'(c_L)} \right) \quad \text{and} \quad \frac{\frac{\partial U^H}{\partial c_{NL}}}{\frac{\partial U^H}{\partial c_L}} < \frac{\frac{\partial U^L}{\partial c_{NL}}}{\frac{\partial U^L}{\partial c_L}}$$

since  $p_L^*(c_{NL}, c_L) < p_H^*(c_{NL}, c_L)$  at all points, by  $\frac{\partial h}{\partial p} < 0$ . Since the  $L$  type's indifference curve is steeper than the  $H$  type's indifference curve at all points, any  $L$  type's indifference curve can cross any  $H$  type's indifference curve at most once, from above to below. Thus the single crossing property is satisfied.  $\square$

**Lemma 7.** *The profit function  $\pi_i(c_{NL}, c_L)$  is decreasing in  $c_L$ , for  $i = H, L$ .*

*Proof.* An increase in  $c_L$  will affect profit in two ways: (1) a direct effect on the amount of indemnity paid, and (2) an indirect effect on the agent's risk level. Both effects are negative so profit is negatively related to  $c_L$ .

Taking the partial derivative of  $\pi_i$  with respect to  $c_L$ , we have

$$\frac{\partial \pi_i}{\partial c_L} = -p_i - \frac{\partial p_i}{\partial c_L} (c_L - c_{NL} + D). \quad (12)$$



The first-order condition of the utility maximization gives  $\frac{\partial h}{\partial p} = u(c_L) - u(c_{NL})$ . If  $c_L$  increases, both  $\frac{\partial h}{\partial p}$  and  $p$  increase because  $\frac{\partial h^2}{\partial p^2} > 0$ . Thus  $\frac{\partial p}{\partial c_L} > 0$ . Notice that  $(c_L - c_{NL} + D)$  is positive. As a result  $\pi_i(c_{NL}, c_L)$  is decreasing in  $c_L$ .  $\square$

Step 1(i): We show that  $(IR_H)$  can be dropped.  $(IR_L)$  implies that the  $L$ -type contract will lie between the vertical line  $c_{NL} = W$  and the  $L$  type's indifference curve that passes through the endowment. By the single crossing property, the  $L$ -type contract must lie above the  $H$  type's indifference curve that passes through the endowment.  $(IC_H)$  implies that  $U^H(c_{NL}^H, c_L^H) \geq U^H(c_{NL}^L, c_L^L) \geq \bar{U}^H$ . Hence  $(IR_L)$ ,  $(IC_H)$ , and the single crossing property together imply  $(IR_H)$ . Thus  $(IR_H)$  is redundant.

Step 1(ii): Next, to show that  $(IR_L)$  can be replaced by  $(\overline{IR}_L)$ , it is useful to consider when pooling happens if  $(IC_H)$  binds.

**Lemma 8.** *If  $(IC_H)$  binds, then  $(IC_L)$  binds if and only if there is a pooling contract, i.e., the contracts for both types are identical.*

*Proof.*  $(\Rightarrow)$  Since  $(IC_H)$  binds, the  $L$ -type contract has to lie on the same  $H$  type's indifference curve as the  $H$ -type contract does.  $(IC_L)$  binding and the single crossing property imply that both contracts are at the intersection of the  $H$  type's and the  $L$  type's indifference curve and there is only one such point. Thus there is a pooling contract.

$(\Leftarrow)$  If there is a pooling contract,  $(IC_L)$  binds.  $\square$

With Lemma 8, the following corollary is immediate.

**Corollary 1.** *If  $(IC_H)$  binds, there is separation if and only if  $(IC_L)$  is slack.*

We will show that  $(IR_L)$  must bind by way of contradiction. Suppose that  $(IR_L)$  does not bind in the solution and let  $(c_{NL}^L, c_L^L)$  and  $(c_{NL}^H, c_L^H)$  be the contracts offered to the  $L$  type and the  $H$  type in the solution respectively. We will consider three cases depending on whether  $(IC_H)$  and  $(IC_L)$  bind: (1)  $(IC_H)$  is slack, (2) both  $(IC_H)$  and  $(IC_L)$  bind and (3)  $(IC_H)$  binds but  $(IC_L)$  is slack.

Suppose  $(IC_H)$  is slack in the solution. Consider a new contract  $(c_{NL}^H, c_L^H - \epsilon)$  for the  $H$  type, where  $\epsilon$  is small enough such that  $U^H(c_{NL}^H, c_L^H - \epsilon) \geq U^H(c_{NL}^L, c_L^L)$ .  $(IC_L)$  holds because  $U^L(c_{NL}^L, c_L^L) \geq U^L(c_{NL}^H, c_L^H) \geq U^H(c_{NL}^H, c_L^H) > U^H(c_{NL}^H, c_L^H - \epsilon)$ . With no change in the contract offered to the  $L$  type,  $(IR_L)$  holds and  $\pi_L$  remains unchanged. By Lemma 7,  $\pi_H$  will increase and so will the total profit  $\pi$ .

Suppose both  $(IC_H)$  and  $(IC_L)$  bind. By Lemma 8, both types purchase the same contract. In this case, a new pooling contract  $(c_{NL}^H, c_L^H - \epsilon)$  such that  $U^L(c_{NL}^H, c_L^H - \epsilon) \geq U^L(W, W - D)$ , for  $\epsilon$  small enough, can be offered. The constraints  $(IR_L)$ ,  $(IC_L)$  and  $(IC_H)$  are satisfied. By Lemma 7,  $\pi_L$  and  $\pi_H$  will increase when this new pooling contract is offered and  $\pi$  will increase as a result.

Suppose  $(IC_H)$  binds but  $(IC_L)$  is slack. Consider a new  $H$ -type contract  $(c_{NL}^H, c_L^H - \epsilon)$  and a new  $L$ -type contract  $(c_{NL}^L, c_L^L - \delta)$ , such that  $U^L(c_{NL}^L, c_L^L - \delta) \geq \bar{U}^L$  and  $U^H(c_{NL}^H, c_L^H) - U^H(c_{NL}^H, c_L^H - \epsilon) = U^H(c_{NL}^L, c_L^L) - U^H(c_{NL}^L, c_L^L - \delta)$ . Then  $U^H(c_{NL}^H, c_L^H - \epsilon) = U^H(c_{NL}^L, c_L^L - \delta)$  for  $\delta$  and  $\epsilon$  small enough. The single crossing property implies that  $U^L(c_{NL}^L, c_L^L - \delta) > U^L(c_{NL}^H, c_L^H - \epsilon)$ . Hence  $(IR_L)$ ,  $(IC_H)$  and  $(IC_L)$  are all satisfied. As both of the new contracts involve a decrease in  $c_L$ , both  $\pi_L$  and  $\pi_H$  will increase and hence  $\pi$  will increase.

We have reached a contradiction under all three cases, and thus  $(IR_L)$  must bind in the solution.

Step 1(iii): We now show that we can replace  $(IC_H)$  with  $(\bar{IC}_H)$  in the problem. Let  $(c_{NL}^L, c_L^L)$  and  $(c_{NL}^H, c_L^H)$  be the optimal contracts offered to the  $L$  type and the  $H$  type respectively.

If the  $H$  type gets  $\bar{U}^H$  in the solution, i.e., zero rent, then the  $H$  type's indifference curve intersects the  $L$  type's indifference curve at the endowment point  $(W, W - D)$ , since  $(IR_L)$  must bind. As  $(W, W - D)$  is the only point on the  $L$  type's indifference curve that involves nonnegative premium and satisfies  $(IC_H)$ , we have  $(c_{NL}^L, c_L^L) = (W, W - D)$ . It follows that  $(IC_H)$  binds.

Suppose that the  $H$  type gets positive rents, i.e.,  $U^H(c_{NL}^H, c_L^H) > \bar{U}^H$ . For a contradiction we will assume that  $(IC_H)$  does not bind in the solution. The  $L$ -type contract must lie strictly below the  $H$  type's indifference curve passing through  $(c_{NL}^H, c_L^H)$ . If the  $L$ -type contract is kept unchanged and a new  $H$ -type contract  $(c_{NL}^H, c_L^H - \epsilon)$  such that  $U^H(c_{NL}^H, c_L^H - \epsilon) \geq U^H(c_{NL}^L, c_L^L)$  for  $\epsilon$  small enough, is offered, then  $(IC_H)$ ,  $(IC_L)$  and  $(\bar{IR}_L)$  are satisfied, as  $U^L(c_{NL}^L, c_L^L) \geq U^L(c_{NL}^H, c_L^H) > U^L(c_{NL}^H, c_L^H - \epsilon)$ . The profit  $\pi_H$  and  $\pi$  will increase when this new set of contracts is offered, and we have a contradiction. Hence  $(IC_H)$  must bind in the solution.

Step 1(iv): The last part of step 1 is to show that  $(IC_L)$  can be replaced by the monotonicity constraint  $c_{NL}^H \leq c_{NL}^L$ . Let  $(c_{NL}^L, c_L^L)$  and  $(c_{NL}^H, c_L^H)$  be the contracts offered to the  $L$  type and the  $H$  type in the solution respectively.

We will first prove that  $(IC_L)$  and  $(IC_H)$  together imply monotonicity. By step 1(iii),  $(IC_H)$  binds and the  $H$  type's indifference curve will cross the  $L$  type's indifference curve exactly once at  $(c_{NL}^L, c_L^L)$ .  $(IC_L)$  and the single crossing property imply that  $(c_{NL}^H, c_L^H)$  has to lie on or to the left of the  $L$ -type contract. Therefore, monotonicity has to be satisfied.

Now, we will prove the reverse direction, i.e.,  $(IC_H)$  and monotonicity imply  $(IC_L)$ . Both  $(IC_H)$  binding and monotonicity imply that  $(c_{NL}^H, c_L^H)$  has to be on or to the left of the intersection between the  $H$  type's indifference curve that passes through  $(c_{NL}^H, c_L^H)$  and the  $L$  type's indifference curve that passes through the endowment. The single crossing property implies that  $(IC_L)$  is satisfied.

We have shown that we can simplify the set of constraints, and in particular problem

(1) is equivalent to problem (6). Next we will decompose the simplified maximization problem (6) into an outer and inner problem by introducing a new variable and solve the inner problem.

**Step 2:** We consider the following problem:

$$\begin{aligned}
\max_R \max_{c_{NL}^H, c_L^H, c_{NL}^L, c_L^L} \quad & \lambda \pi_L(c_{NL}^L, c_L^L) + (1 - \lambda) \pi_H(c_{NL}^H, c_L^H) & (7) \\
\text{subject to} \quad & U^L(c_{NL}^L, c_L^L) = \bar{U}^L & (\overline{IR}_L) \\
& U^H(c_{NL}^H, c_L^H) = U^H(c_{NL}^L, c_L^L) & (\overline{IC}_H) \\
& c_{NL}^H \leq c_{NL}^L & (\text{monotonicity}) \\
& U^H = \bar{U}^H + R, & (\bar{R})
\end{aligned}$$

where  $R$  is the amount of rents given to  $H$  types and  $R \geq 0$ .

The inner problem, denoted as  $ALT(R)$ , pins down the set of profit-maximizing contracts that satisfies  $(\overline{IR}_L)$ ,  $(\overline{IC}_H)$ , monotonicity and an additional constraint  $(\bar{R})$ . The constraint  $(\bar{R})$  specifies the utility of  $H$  types in the solution to be  $\bar{U}^H + R$ . The outer problem is to choose  $R$  to maximize profits. It is clear that the solution to (6) is a solution to the combined inner and outer problem.

Indeed, Assumption 1 ensures that the solution to  $ALT(R)$  is unique for any  $R$ . To see this, the constraints  $(\overline{IR}_L)$  and  $(\bar{R})$  specify exactly the values of  $U^H$  and  $U^L$  in the solution and thus the corresponding  $H$  type's and  $L$  type's indifference curves.  $(\overline{IC}_H)$  locates the  $L$ -type contract at the intersection of the two indifference curves. The insurer will then choose a contract on the  $H$  type's indifference curve that maximizes her profits from the  $H$  type subject to monotonicity. By Assumption 1, the solution exists and is unique.

Recall  $(c_{NL,R}^{H*}, c_{L,R}^{H*})$  is the  $H$ -type first-best contract associated with rent  $R$ . Before characterizing the solution, it is helpful to define a few terms. Let  $(c_{NL,R}^L, c_{NL,R}^L)$  and  $(c_{NL,R}^H, c_{NL,R}^H)$  be the optimal  $L$ -type and  $H$ -type contracts in the solution to  $ALT(R)$  respectively. By Assumption 1, the upward sloping nature of  $\vec{c}^H(R)$  guarantees that there is only one  $R$  such that the  $H$ -type first-best-contract locus  $\vec{c}^H(R)$  intersects the  $L$  type's indifference curve. Let  $R^L$  be the associated level of rent and  $(c_{NL,R^L}^{H*}, c_{L,R^L}^{H*})$  be their intersection. Let  $(\bar{c}_{NL}, \bar{c}_L)$  be the intersection of the  $L$  type's indifference curve and the 45-degree line. Let  $\bar{R}$  be the rent such that  $U^H(\bar{c}_{NL}, \bar{c}_L) - \bar{U}^H = \bar{R}$ .

We are reading to characterize the solution to  $ALT(R)$  in the following lemma:

**Lemma 9.** *The solution to  $ALT(R)$  can be characterized as follows.*

1. If  $0 \leq R \leq R_L$ , the  $H$ -type contract is first-best with rent  $R$ ; the  $L$ -type contract provides the minimum utility to  $L$  types and  $\bar{U}^H + R$  to  $H$  types.

2. If  $R_L \leq R \leq \bar{R}$ , then  $(c_{NL,R}^L, c_{NL,R}^L) = (c_{NL,R}^H, c_{NL,R}^H)$ ; the pooling contract provides the minimum utility to  $L$  types and  $\bar{U}^H + R$  to  $H$  types.
3. If  $R > \bar{R}$ , there is no solution to  $ALT(R)$ .

*Proof.* 1. For  $0 \leq R \leq R^L$ , the  $H$ -type first-best contract associated with  $R$  maximizes profits on the  $H$  type and satisfies the monotonicity constraint. Thus  $c_{NL,R}^H = c_{NL,R}^{H*}$ . The constraint  $(\bar{IR}_L)$  implies that the  $L$ -type contract provides zero utility rents to the  $L$  type.  $(\bar{IC}_H)$  implies that  $U^H(c_{NL,R}^L, c_{NL,R}^L) = \bar{U}^H + R$ . The upward sloping nature of  $\vec{c}^H(R)$  implies that we have separating contracts if  $0 \leq R < R^L$ .

2. If  $R = R^L$ , then the  $H$ -type first-best contract  $(c_{NL,R^L}^{H*}, c_{L,R^L}^{H*})$  can be offered to both types.

If  $R^L < R \leq \bar{R}$ , the  $H$ -type first-best-contract locus  $\vec{c}^H(R)$  lies to the right of the  $L$  type's indifference curve, so by monotonicity and  $(\bar{IC}_H)$ , the  $H$ -type contract cannot lie on the first-best-contract locus  $\vec{c}^H(R)$ . We claim that the pooling contract  $(c_{NL,R}^H, c_{L,R}^H) = (c_{NL,R}^L, c_{L,R}^L)$  at the intersection between the  $H$  type's indifference curve with rent  $R$  and the  $L$  type's indifference curve that passes through the endowment maximizes profits. Notice that by Assumption 1, any contract that lies on the left of the pooling contract  $(c_{NL,R}^H, c_{L,R}^H)$  will yield lower profits from  $H$  types. Thus the insurer will choose to offer the pooling contract as described above to both types.

3. For any  $R > \bar{R}$ , the  $H$  type's indifference curve will not intersect the  $L$  type's indifference curve passing the endowment, so  $(\bar{IC}_H)$  cannot be satisfied for any choices of contracts. Thus no solution exists.

□

**Step 3:** Now that we have characterized the solution to the inner problem  $ALT(R)$ , we proceed to solve the outer problem. Define  $\hat{\pi}_i(R)$  to be the profits from type  $i$  in the solution to  $ALT(R)$  when the rent given to  $H$  types is  $R$ .

The outer problem can be written as

$$\max_R \pi(R, \lambda) = \lambda \hat{\pi}_L(R) + (1 - \lambda) \hat{\pi}_H(R) \quad (13)$$

$$\text{subject to } 0 \leq R \leq \bar{R}. \quad (14)$$

Since  $\hat{\pi}_L(R)$  and  $\hat{\pi}_H(R)$  are continuous,  $\pi(R, \lambda)$  is also continuous. Define

$$R^*(\lambda) = \arg \max_{R \in [0, \bar{R}]} \pi(R, \lambda).$$

By the Theorem of the Maximum,  $R^*(\lambda)$  is nonempty for all  $\lambda$  and is upper hemicontinuous. Let  $R^M$  be the rent such that  $U^H(c_{NL}^{L*}, c_L^{L*}) = \bar{U} + R^M$ , where  $(c_{NL}^{L*}, c_L^{L*}) =$

$\arg \max_{U^L(c_{NL}, c_L)=\bar{U}} \pi_L$  is the  $L$  type's first-best zero-rent contract.<sup>14</sup> The following lemma shows how profits from each type changes with rents to the  $H$  type.

**Lemma 10.**  $\widehat{\pi}_L(R)$  is nondecreasing for  $0 \leq R \leq R^M$  and non-increasing for  $R^M \leq R \leq \bar{R}$ .  $\widehat{\pi}_H(R)$  is decreasing for  $0 \leq R \leq \bar{R}$ .

*Proof.* By Assumption 1,  $\widehat{\pi}_L(R)$  is nondecreasing for  $0 \leq R \leq R^M$  and non-increasing for  $R^M \leq R \leq \bar{R}$ .

We show  $\widehat{\pi}_H(R)$  is decreasing in  $R$  by contradiction. Suppose that there exists  $R' > R$  such that  $\widehat{\pi}_H(R') \geq \widehat{\pi}_H(R)$ . Let  $(c_{NL,R}^H, c_{NL,R}^H)$  and  $(c_{NL,R}^L, c_{NL,R}^L)$  be the contracts offered to  $H$  types and  $L$  types respectively in the solution to  $ALT(R)$ . We consider two cases: (1)  $(c_{NL,R}^H, c_{NL,R}^H)$  is first-best and (2)  $(c_{NL,R}^H, c_{NL,R}^H)$  is not first-best.

Consider  $(c_{NL,R'}^H, c_{NL,R'}^H - \epsilon)$  such that  $U^H(c_{NL,R'}^H, c_{NL,R'}^H - \epsilon) = U^H(c_{NL,R}^H, c_{NL,R}^H)$ . If  $(c_{NL,R}^H, c_{NL,R}^H)$  is first-best, then  $\pi_H(c_{NL,R'}^H, c_{NL,R'}^H - \epsilon) > \widehat{\pi}_H(R') \geq \widehat{\pi}_H(R)$ , where the first inequality follows from Lemma 7, violates that  $(c_{NL,R}^H, c_{NL,R}^H)$  is the first-best contract associated with  $R$ . If  $(c_{NL,R}^H, c_{NL,R}^H)$  is not first-best, we must have  $R' > R \geq R_L$  by Lemma 9 and thus  $c_{NL,R'}^H < c_{NL,R}^H$ . The inequality  $\pi_H(c_{NL,R'}^H, c_{NL,R'}^H - \epsilon) > \widehat{\pi}_H(R') \geq \widehat{\pi}_H(R)$  implies that  $(c_{NL,R'}^H, c_{NL,R'}^H - \epsilon)$  lies on the left of  $(c_{NL,R}^H, c_{NL,R}^H)$  on the same  $H$  type's indifference curve and yields higher profits. This violates that  $(c_{NL,R}^H, c_{NL,R}^H)$  is a solution to  $ALT(R)$ .  $\square$

It follows that we can restrict our attention to rents not greater than  $R^M$ .

**Corollary 2.** If  $R > R^M$ , then  $R \notin R^*(\lambda)$  for any  $0 \leq \lambda \leq 1$ .

*Proof.* Suppose, for a contradiction, that there exists  $R > R^M$  such that  $R \in R^*(\lambda)$  for some  $\lambda$ . Then by optimality of  $R$ , we have

$$\lambda \widehat{\pi}_L(R) + (1 - \lambda) \widehat{\pi}_H(R) \geq \lambda \widehat{\pi}_L(R^M) + (1 - \lambda) \widehat{\pi}_H(R^M).$$

Rearranging, we get

$$\lambda(\widehat{\pi}_L(R) - \widehat{\pi}_L(R^M)) + (1 - \lambda)(\widehat{\pi}_H(R) - \widehat{\pi}_H(R^M)) \geq 0. \quad (15)$$

By Lemma 10,  $\widehat{\pi}_L(R) \leq \widehat{\pi}_L(R^M)$  and  $\widehat{\pi}_H(R) < \widehat{\pi}_H(R^M)$ , and this contradicts the inequality (15). Thus  $R \notin R^*(\lambda)$ .  $\square$

This proves that the constraint (14) can be replaced by  $0 \leq R \leq R^M$ . In other words, any  $R > R^M$  is never an optimal value of rents given to  $H$  types to maximize profits, so we can narrow down the range of rents to be considered in our analysis.

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<sup>14</sup>Due to moral hazard, we have  $R^M < \bar{R}$

Before solving the outer problem, it is useful to consider the optimal rent when there is only one type in the market, i.e.,  $\lambda = 0$  or  $\lambda = 1$ . When  $\lambda = 0$ , we have  $\pi(R, 0) = \widehat{\pi}_H(R)$ . Since there are only  $H$  types in the market, this is equivalent to the first-best problem of the  $H$  type. It is obvious that  $R^*(0) = \{0\}$ . When  $\lambda = 1$ , we have  $\pi(R, 1) = \widehat{\pi}_L(R)$ . Since there are only  $L$  types in the market, this is equivalent to the first-best problem of the  $L$  type. Thus  $R^*(1) = \{R^M\}$ .

To study the comparative statics of  $R^*$  with respect to  $\lambda$ , we have to consider two cases: (1)  $R^M < R^L$  and (2)  $R^M \geq R^L$ .

The following lemmas show that it is possible to relate  $\lambda$  to  $R^*$  and the comparative statics of the inner problem can be carried over.

**Lemma 11.**  $\pi(R, \lambda)$  satisfies increasing differences in  $(R, \lambda)$ .

*Proof.* Let  $R' > R$  and  $\lambda' > \lambda$ .

Then

$$\pi(R', \lambda') - \pi(R', \lambda) = (\lambda' - \lambda)\widehat{\pi}_L(R') + (\lambda - \lambda')\widehat{\pi}_H(R')$$

and

$$\pi(R, \lambda') - \pi(R, \lambda) = (\lambda' - \lambda)\widehat{\pi}_L(R) + (\lambda - \lambda')\widehat{\pi}_H(R).$$

By Lemma 10,  $\widehat{\pi}_L$  is nondecreasing in  $R$  and  $\widehat{\pi}_H$  is decreasing in  $R$ . So

$$\begin{aligned}\pi(R', \lambda') - \pi(R', \lambda) &> \pi(R, \lambda') - \pi(R, \lambda) \text{ and} \\ \pi(R', \lambda') - \pi(R, \lambda') &> \pi(R', \lambda) - \pi(R, \lambda).\end{aligned}$$

□

By Topkis's theorem,  $R^*(\lambda)$  is non-decreasing in  $\lambda$  in the strong set order. We will prove some properties of  $R^*$  that are useful for doing comparative statics in the following lemma.

**Lemma 12.**  $R^*$  satisfies the following properties:

1. For any  $R \in (0, R^M)$  and  $\lambda \neq \lambda'$ ,  $R \notin R^*(\lambda) \cap R^*(\lambda')$ ;
2.  $s(\lambda) = \sup\{R^*(\lambda)\}$  is nondecreasing in  $\lambda$ ;
3.  $g(\lambda) = \inf\{R^*(\lambda)\}$  is nondecreasing in  $\lambda$ ;
4. If  $\lambda' > \lambda$ , then  $h(\lambda') \geq s(\lambda)$ .

*Proof.* 1. For a contradiction, suppose that  $R \in R^*(\lambda_1) \cap R^*(\lambda_2)$  where  $\lambda_1 < \lambda_2$ . We will look into two cases: (i)  $R < R^L$  and (ii)  $R \geq R^L$ . Let

$$\begin{aligned}\mathcal{L} &= \lambda_1 \pi_L + (1 - \lambda_1) \pi_H + \alpha (U^L(c_{NL}^L, c_L^L) - U^L(W, W - D)) \\ &\quad + \beta (U^H(c_{NL}^H, c_L^H) - U^H(c_{NL}^L, c_L^L)) + \gamma (c_{NL}^L - c_{NL}^H)\end{aligned}$$

be the Lagrangian for problem (6), where  $\alpha$ ,  $\beta$  and  $\gamma$  are the Lagrange multipliers associated with  $(\overline{IR_L})$ ,  $(\overline{IC_H})$  and the monotonicity constraint respectively.

For  $R < R^L$ , let  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  be the optimal contracts associated with  $R$  for  $H$  types and  $L$  types respectively. Note that since monotonicity does not bind for this allocation, the multiplier  $\gamma = 0$ . For a small change  $\Delta R > 0$ , the new optimal contracts will be  $(c_{NL}^H + \Delta_{NL}^H, c_L^H + \Delta_L^H)$  and  $(c_{NL}^L + \Delta_{NL}^L, c_L^L + \Delta_L^L)$ . Taking a directional derivative of  $\mathcal{L}$  in the direction  $\delta = (\Delta_{NL}^H, \Delta_L^H, \Delta_{NL}^L, \Delta_L^L)$ , we have

$$\begin{aligned} & \nabla_\delta \mathcal{L}(c_{NL}^H, c_L^H, c_{NL}^L, c_L^L) \\ &= \lambda_1 \nabla_\delta \pi_L + (1 - \lambda_1) \nabla_\delta \pi_H + \alpha \nabla_\delta (U^L(c_{NL}^L, c_L^L) - U^L(W, W - D)) \\ & \quad + \beta \nabla_\delta (U^H(c_{NL}^H, c_L^H) - U^H(c_{NL}^L, c_L^L)) + \gamma \nabla_\delta (c_{NL}^L - c_{NL}^H) \\ &= \lambda_1 \nabla_\delta \pi_L + (1 - \lambda_1) \nabla_\delta \pi_H \\ &= 0. \end{aligned}$$

The last three terms are zero because  $(\overline{IC_H})$  and  $(\overline{IR_L})$  are satisfied and  $\gamma = 0$ . Since  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  is a solution to the problem (6) for  $\lambda_1$ , the directional derivative equals zero. Since  $R$  is also a maximizer for  $\lambda_2$ , we have

$$\lambda_2 \nabla_\delta \pi_L + (1 - \lambda_2) \nabla_\delta \pi_H = 0.$$

By Lemma 10,  $\nabla_\delta \pi_L$  is non-negative since the change in profits associated with this increase in rent is non-negative, while  $\nabla_\delta \pi_H$  is negative since the change in profits associated with this increase in rent is negative. There can only be one  $\lambda$  that satisfies the equality, and we have reached a contradiction.

For  $R \geq R^L$ , let  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  be the optimal contracts associated with  $R$  for  $H$  types and  $L$  types respectively. By Lemma 9, the two contracts are identical. For a small change  $\Delta R > 0$ , the new optimal contracts will be  $(c_{NL}^H + \Delta_{NL}^H, c_L^H + \Delta_L^H)$  and  $(c_{NL}^L + \Delta_{NL}^L, c_L^L + \Delta_L^L)$ . Taking a directional derivative of  $\mathcal{L}$  along the direction  $\delta = (\Delta_{NL}^H, \Delta_L^H, \Delta_{NL}^L, \Delta_L^L)$ , we have

$$\begin{aligned} & \nabla_\delta \mathcal{L}(c_{NL}^H, c_L^H, c_{NL}^L, c_L^L) \\ &= \lambda_1 \nabla_\delta \pi_L + (1 - \lambda_1) \nabla_\delta \pi_H + \alpha \nabla_\delta (U^L(c_{NL}^L, c_L^L) - U^L(W, W - D)) \\ & \quad + \beta \nabla_\delta (U^H(c_{NL}^H, c_L^H) - U^H(c_{NL}^L, c_L^L)) + \gamma \nabla_\delta (c_{NL}^L - c_{NL}^H) \\ &= \lambda_1 \nabla_\delta \pi_L + (1 - \lambda_1) \nabla_\delta \pi_H \\ &= 0. \end{aligned}$$

The last three terms are zero because  $(\overline{IR_L})$  and  $(\overline{IC_H})$  are satisfied and mono-

tonicity binds. Since  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  is a solution to the problem (6) for  $\lambda_1$ , the directional derivative equals zero. Since  $R$  is also a maximizer for  $\lambda_2$ ,

$$\lambda_2 \nabla_{\delta} \pi_L + (1 - \lambda_2) \nabla_{\delta} \pi_H = 0.$$

Similarly, there are no two distinct values of  $\lambda$  that satisfy the two equalities, thus we have a contradiction.

2. True by Topkis's Theorem.
3. True by Topkis's Theorem.
4. If  $\lambda = 0$ , the  $s(\lambda) = 0$  and it is clear that  $g(\lambda') \geq s(\lambda)$ .

Suppose  $\lambda \neq 0$ . We prove this by contradiction. Suppose there exists  $\lambda' > \lambda$  such that  $g(\lambda') < s(\lambda)$ . Optimality of  $s(\lambda)$  implies

$$\lambda \widehat{\pi}_L(s(\lambda)) + (1 - \lambda) \widehat{\pi}_H(s(\lambda)) \geq \lambda \widehat{\pi}_L(g(\lambda')) + (1 - \lambda) \widehat{\pi}_H(g(\lambda')).$$

Then

$$\frac{\widehat{\pi}_L(s(\lambda)) - \widehat{\pi}_L(g(\lambda'))}{\widehat{\pi}_H(g(\lambda')) - \widehat{\pi}_H(s(\lambda))} \geq \frac{1 - \lambda}{\lambda} > \frac{1 - \lambda'}{\lambda'}.$$

Rearranging the equation gives

$$\lambda' \widehat{\pi}_L(s(\lambda)) + (1 - \lambda') \widehat{\pi}_H(s(\lambda)) > \lambda' \widehat{\pi}_L(g(\lambda')) + (1 - \lambda') \widehat{\pi}_H(g(\lambda')).$$

This implies  $g(\lambda') \notin R^*(\lambda')$  and we have arrived at a contradiction. Thus  $g(\lambda') \geq s(\lambda)$  for all  $\lambda' > \lambda$ . □

**Lemma 13.** *There exists  $\underline{\lambda}$  such that*

1.  $0 \in R^*(\underline{\lambda})$ ;
2. If  $\lambda > \underline{\lambda}$ , then  $0 \notin R^*(\lambda)$ ;
3. If  $\lambda < \underline{\lambda}$ , then  $R^*(\lambda) = \{0\}$ .

*Proof.* 1. Let  $\underline{T} = \{\lambda : 0 \in R^*(\lambda)\}$ .  $\underline{T}$  is nonempty because  $0 \in \underline{T}$  and is bounded above by 1. By completeness of  $\mathbb{R}$ ,  $\sup \underline{T}$  exists. Let  $\underline{\lambda} = \sup \underline{T}$ . Upper hemicontinuity implies  $\underline{\lambda} \in \underline{T}$ .

2. If  $\lambda > \underline{\lambda}$ , we have  $0 \notin R^*(\lambda)$  by definition of supremum.



3. From part (1),  $0 \in R^*(\underline{\lambda})$  implies  $g(\underline{\lambda}) = 0$ . Lemma 12 implies that if  $\lambda < \underline{\lambda}$ , then  $g(\lambda) \leq s(\lambda) \leq g(\bar{\lambda}) = 0$ . Thus  $g(\lambda) = s(\lambda) = 0$  and  $R^*(\lambda) = \{0\}$ .

□

**Lemma 14.** *Suppose  $R^M > R^L > 0$ . There exists  $\bar{\lambda}$  such that*

1. *If  $\lambda < \bar{\lambda}$ , then  $R^L \notin R^*(\lambda)$ ;*
2. *If  $\lambda > \bar{\lambda}$ , then  $R^*(\lambda) \subseteq [R^L, R^M]$ .*

*Proof.* Let  $\bar{T} = \{\lambda : s(\lambda) \leq R_L\}$ .  $\bar{T}$  is nonempty, because  $0 \in \bar{T}$ .  $\bar{T}$  is bounded above by 1, so  $\sup \bar{T}$  exists. Let  $\bar{\lambda} = \sup \bar{T}$ .

1. We will consider two cases: (i)  $\bar{\lambda} \in \bar{T}$  and (ii)  $\bar{\lambda} \notin \bar{T}$ .

Suppose  $\bar{\lambda} \in \bar{T}$ . If  $s(\bar{\lambda}) = 0$ ,  $\bar{\lambda} \leq \underline{\lambda}$  and it is clear that  $R^L \notin R^*(\lambda)$  for  $\lambda < \bar{\lambda}$ . For  $s(\bar{\lambda}) > 0$ , by Lemma 12 we have  $s(\lambda) < s(\bar{\lambda}) \leq R^L$  for all  $\lambda < \bar{\lambda}$ .

For  $\bar{\lambda} \notin \bar{T}$ , suppose for a contradiction that there exists  $\lambda < \bar{\lambda}$  such that  $R^L \in R^*(\lambda)$ . Note that  $\lambda$  is necessarily an upper bound for  $\bar{T}$ , since by Lemma 12, we have  $s(\lambda') > s(\lambda) \geq R^L$  if  $\lambda' \in (\lambda, \bar{\lambda})$ . We have a contradiction.

2. By definition of  $\bar{T}$ , if  $\lambda > \bar{\lambda}$ , then  $g(\lambda) \geq s(\lambda - \epsilon) > R_L$  for some  $\epsilon$  such that  $\lambda - \epsilon > \bar{\lambda}$ . Thus  $R^*(\lambda) \subseteq [R^L, R^M]$ .

Hence our choice of  $\bar{\lambda}$  is  $\sup \bar{T}$  satisfies all the conditions.

□

Finally we can characterize the optimal contracts in the two cases: (1)  $R^M < R^L$  and (2)  $R^M \geq R^L$ .

Case (1) is straightforward – since the possible range of  $R$  is from 0 to  $R^M$ , then  $R^M < R^L$  means that the optimal rent must be less than  $R^M$ . By Lemma 13, there exists  $\underline{\lambda} \geq 0$  such that  $R^*(\lambda) = \{0\}$  if  $\lambda < \underline{\lambda}$ , meaning that only  $H$  types purchase the first-best contract with zero utility rents and  $L$  types do not participate in the market. If  $\lambda > \underline{\lambda}$ , then  $H$  types purchase the first-best contract with a positive rent while  $L$  types purchase a zero-rent contract. In this case,  $\bar{\lambda} > 1$  and there will be no pooling contracts.

For case (2), if  $R^M \geq R^L$ , Lemma 13 gives  $\underline{\lambda}$  such that when  $0 \leq \lambda < \underline{\lambda}$  only  $H$  types purchase the first-best contract and  $L$  types do not. Lemma 14 gives  $\bar{\lambda}$ . If  $\underline{\lambda} < \lambda < \bar{\lambda}$ , the  $H$ -type contract will be first-best with positive rents and the  $L$ -type contract gives zero rent to  $L$  types. If  $\lambda \geq \bar{\lambda}$ , the optimal rent will be no less than  $R^L$ , the minimum level of rent for a pooling contract. Thus the  $H$ -type and  $L$ -type contracts are the same.

## C Proof of Lemma 1

Suppose  $(c_{NL}, c_L)$  and  $\bar{p}$  is a solution to the optimization problem (10). Consider a small movement of the contract to  $(c'_{NL}, c'_L)$  in the direction  $\delta$ , holding utility and  $\bar{p}$  constant, that results in a decrease in  $p^*$ , i.e.,  $p^*(c'_{NL}, c'_L) < p^*(c_{NL}, c_L)$ . Graphically, this corresponds to an inward movement of the contract along the indifference curve towards the endowment point. The change in the value of the Lagrangian is

$$\begin{aligned}\nabla_\delta \mathcal{L} &= \nabla_\delta \pi + \alpha \nabla_\delta (U(c_{NL}, c_L; \bar{p}) - U(W, W - D; \bar{p})) + \mu \nabla_\delta (p^*(c_{NL}, c_L; \bar{p}) - \bar{p}) \\ &= \nabla_\delta \pi + \mu \nabla_\delta (p^*(c'_{NL}, c'_L; \bar{p}) - \bar{p}).\end{aligned}$$

The term associated with  $\alpha$  equals zero since  $(IR)$  binds under the movement. The optimality of the solution implies that  $\nabla_\delta \mathcal{L} = 0$ . Rearranging the equation, we have

$$\nabla_\delta \pi = -\mu \nabla_\delta (p^*(c_{NL}, c_L; \bar{p}) - \bar{p}).$$

If  $\mu > 0$ , then  $\nabla_\delta \pi > 0$ . Given a fixed  $\bar{p}$ , this particular movement of contract would be profitable for the insurer. If she could ignore the consistency constraint, she would want to choose a contract that yields a lower  $p^*$  to increase profits. The contract is thus distorted upward. The positive sign of  $\mu$  acts as a penalty for violating the consistency constraint in the way described.

A similar argument can be made to show that the optimal one-type contract with endogeneity is distorted downward if  $\mu < 0$ .

## D Proof of Proposition 2

Recall that the Lagrangian for the simplified problem is

$$\begin{aligned}\mathcal{L}(c_{NL}^L, c_L^L, c_{NL}^H, c_L^H, \bar{p}) &= \lambda \pi_L(c_{NL}^L, c_L^L; \bar{p}) + (1 - \lambda) \pi_H(c_{NL}^H, c_L^H; \bar{p}) \\ &\quad + \alpha (U^L(c_{NL}^L, c_L^L; \bar{p}) - U^L(W, W - D; \bar{p})) \\ &\quad + \beta (U^H(c_{NL}^H, c_L^H; \bar{p}) - U^H(c_{NL}^L, c_L^L; \bar{p})) + \gamma (c_{NL}^L - c_{NL}^H) \\ &\quad + \mu (\lambda p_L^*(c_{NL}^L, c_L^L; \bar{p}) + (1 - \lambda) p_H^*(c_{NL}^H, c_L^H; \bar{p}) - \bar{p}).\end{aligned}\tag{9}$$

First, we look into the case of separating contracts. Note that since monotonicity does not bind, the multiplier  $\gamma$  is zero. Let  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$ , where  $(c_{NL}^H, c_L^H) \neq (c_{NL}^L, c_L^L)$ , be the solution to monopoly's endogenous problem and  $\bar{p}$  be the associated optimal average level of risk.

To evaluate the distortion on the  $H$  type's contract, we take the directional derivative of the Lagrangian (equation 9) in the direction  $\delta = (\Delta c_{NL}^H, \Delta c_L^H, 0, 0)$ , keeping  $\bar{p}$  and  $U^H$

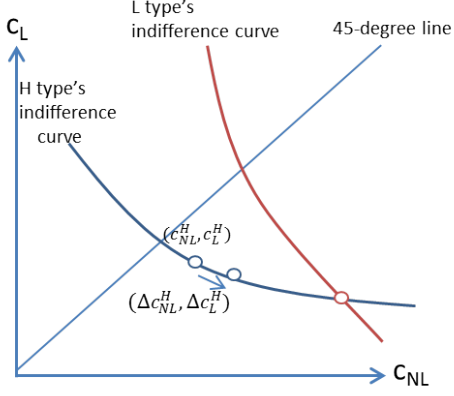


Figure 16:  $\mu$  is positive

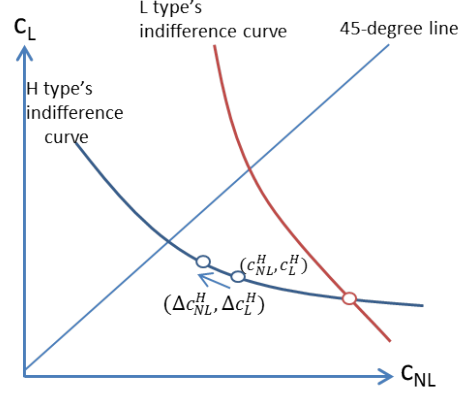


Figure 17:  $\mu$  is negative

constant and  $L$ -type contract unchanged:

$$\begin{aligned}
\nabla_{\delta} \mathcal{L} &= \lambda \nabla_{\delta} \pi_L + (1 - \lambda) \nabla_{\delta} \pi_H + \alpha \cdot 0 + \beta \cdot 0 + \gamma \nabla_{\delta} (c_{NL}^L - c_{NL}^H) \\
&\quad + \mu \nabla_{\delta} (\lambda p_L^*(c_{NL}^L, c_L^L; \bar{p}) + (1 - \lambda) p_H^*(c_{NL}^H, c_L^H; \bar{p}) - \bar{p}) \\
&= (1 - \lambda) \nabla_{\delta} \pi_H + \mu (1 - \lambda) \nabla_{\delta} p_H^* \\
&= 0.
\end{aligned}$$

The terms associated with  $\alpha$  and  $\beta$  are zero because both  $(\overline{IR}_L)$  and  $(\overline{IC}_H)$  are satisfied under the change. Since  $\gamma = 0$ , the term associated with the monotonicity constraint is zero. The  $L$ -type contract remains unchanged, so  $\nabla_{\delta} \pi_L = 0$ . Since  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  is a solution to the endogenous problem, the change in value of the Lagrangian equals zero.

If  $\mu$  is positive, consider an inward movement of the  $H$ -type contract towards the endowment point, i.e.,  $\Delta c_{NL}^H > 0$  and  $\Delta c_L^H < 0$ . Refer to Figure 16. Since  $\nabla_{\delta} p_H^* < 0$ , we have  $(1 - \lambda) \nabla_{\delta} \pi_H > 0$  and the  $H$ -type contract cannot be at the tangency point between the iso-profit curve and the  $H$  type's indifference curve. Thus  $(c_{NL}^H, c_L^H)$  cannot be an optimal  $H$ -type contract in the exogenous problem with  $\bar{p}$ . In particular, the  $H$ -type contract is  $s$ -distorted upward and lies to the left of the tangency point.

If  $\mu$  is negative, consider an outward movement of the  $H$ -type contract away from the endowment point, i.e.,  $\Delta c_{NL}^H < 0$  and  $\Delta c_L^H > 0$ . Refer to Figure 17. Since  $\nabla_{\delta} p_H^* > 0$ , we have  $(1 - \lambda) \nabla_{\delta} \pi_H > 0$  and the  $H$ -type contract cannot be at the tangency point between the iso-profit curve and the  $H$  type's indifference curve. Thus  $(c_{NL}^H, c_L^H)$  cannot be an optimal  $H$ -type contract in the exogenous problem with  $\bar{p}$ . In particular, the  $H$ -type contract is  $s$ -distorted downward and lies to the right of the tangency point.

To evaluate the distortion on the  $L$  type's contract, we will take the directional derivative of the Lagrangian (equation 9) in the direction  $\delta = (\Delta c_{NL}^H, \Delta c_L^H, \Delta c_{NL}^L, \Delta c_L^L)$ , keeping

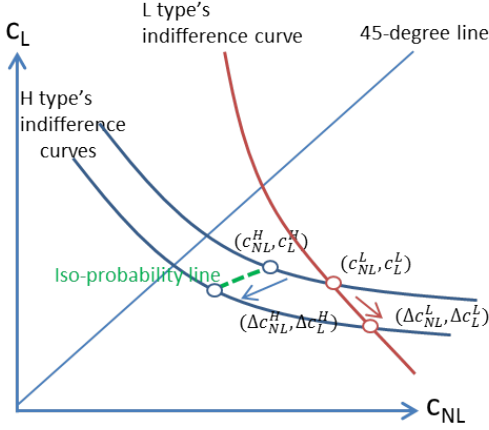


Figure 18:  $\mu$  is positive

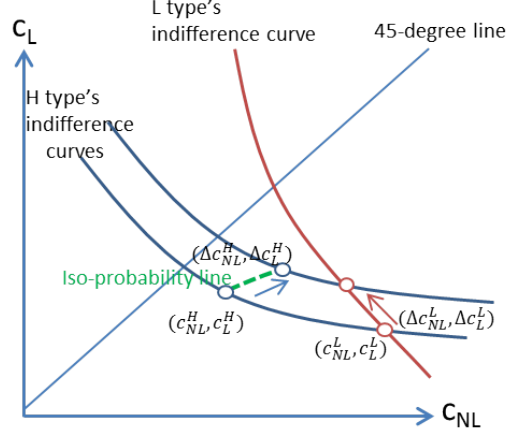


Figure 19:  $\mu$  is negative

$\bar{p}$ ,  $U^L$  and  $p_H^*$  constant and  $\overline{IC}_H$  satisfied, i.e.,

$$\begin{aligned}\nabla_{\delta}\mathcal{L} &= \lambda\nabla_{\delta}\pi_L + (1-\lambda)\nabla_{\delta}\pi_H + \alpha \cdot 0 + \beta \cdot 0 \\ &\quad + \gamma\nabla_{\delta}(c_{NL}^L - c_{NL}^H) + \mu\nabla_{\delta}(\lambda p_L^* + (1-\lambda)p_H^*) \\ &= \lambda\nabla_{\delta}\pi_L + (1-\lambda)\nabla_{\delta}\pi_H + \mu\lambda\nabla_{\delta}p_L^* \\ &= 0.\end{aligned}$$

The terms associated with  $\alpha$  and  $\beta$  are zero because both  $(IR_L)$  and  $(IC_H)$  bind under the change. Since  $\gamma = 0$ , the term associated with the monotonicity constraint equals zero. Since  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  is a solution to the endogenous problem, the change in value of the Lagrangian equals zero.

If  $\mu$  is positive, consider a movement of the  $H$ -type contract down along the iso-probability line to the left and the  $L$ -type contract down along the indifference curve to the right, i.e.  $\Delta c_{NL}^H < 0$ ,  $\Delta c_L^H < 0$ ,  $\Delta c_{NL}^L > 0$  and  $\Delta c_L^L < 0$ . Refer to Figure 18. Under this change,  $\mu\lambda\nabla_{\delta}p_L^* < 0$  and thus  $\lambda\nabla_{\delta}\pi_L + (1-\lambda)\nabla_{\delta}\pi_H > 0$ . This implies that  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  cannot be a solution to the exogenous problem with  $\bar{p}$ . It is clear that the  $L$ -type contract is  $s$ -distorted upward.

If  $\mu$  is negative, consider a movement of the  $H$ -type contract up along the iso-probability line to the right and the  $L$ -type contract up along the indifference curve to the left, i.e.,  $\Delta c_{NL}^H > 0$ ,  $\Delta c_L^H > 0$ ,  $\Delta c_{NL}^L < 0$  and  $\Delta c_L^L > 0$ . Refer to Figure 19. Under this change,  $\mu\lambda\nabla_{\delta}p_L^* < 0$  and thus  $\lambda\nabla_{\delta}\pi_L + (1-\lambda)\nabla_{\delta}\pi_H > 0$ . This implies that  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  cannot be a solution to the exogenous problem with  $\bar{p}$ . It is clear that the  $L$ -type contract is  $s$ -distorted downward.

Second, we look at the distortion on a pooling contract. Let  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$ , where  $(c_{NL}^H, c_L^H) = (c_{NL}^L, c_L^L)$ , be the solution to endogenous problem (8) and  $\bar{p}$  be the associated optimal average level of risk.

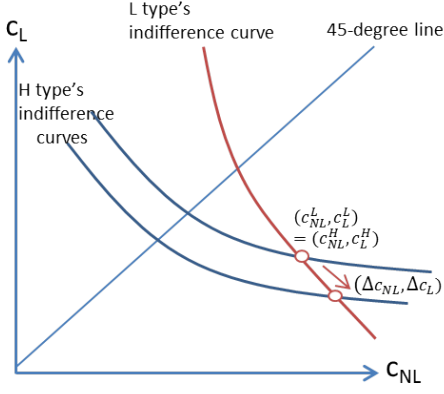


Figure 20:  $\mu$  is positive

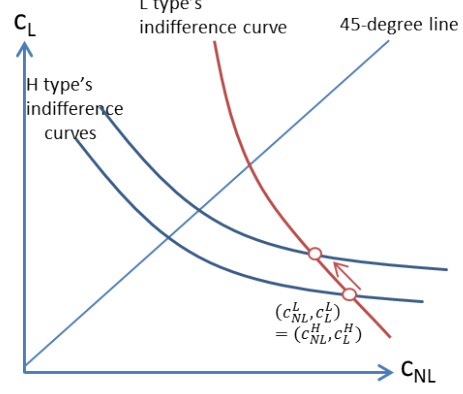


Figure 21:  $\mu$  is negative

We will take the directional derivative of the Lagrangian in the direction  $\delta = (\Delta c_{NL}, \Delta c_L, \Delta c_{NL}, \Delta c_L)$ , keeping  $\bar{p}$  and  $U^L$  constant, i.e.,

$$\begin{aligned}
 \nabla_{\delta} \mathcal{L} &= \lambda \nabla_{\delta} \pi_L + (1 - \lambda) \nabla_{\delta} \pi_H + \alpha \cdot 0 + \beta \cdot 0 + \gamma \nabla_{\delta} (c_{NL} - c_{NL}) \\
 &\quad + \mu \nabla_{\delta} (\lambda \Delta p_L^* + (1 - \lambda) \Delta p_H^*) \\
 &= \lambda \nabla_{\delta} \pi_L + (1 - \lambda) \nabla_{\delta} \pi_H + \mu (\lambda \nabla_{\delta} p_L^* + (1 - \lambda) \nabla_{\delta} p_H^*) \\
 &= 0
 \end{aligned}$$

The terms associated with  $\alpha$  and  $\beta$  are zero because both  $(\overline{IR}_L)$  and  $(\overline{IC}_H)$  are satisfied under the change. Note that monotonicity also binds. Since  $(c_{NL}^H, c_L^H)$  and  $(c_{NL}^L, c_L^L)$  is a solution to the endogenous problem, the change in value of the Lagrangian equals zero.

If  $\mu$  is positive, consider a movement of the pooling contract down along the  $L$  type's indifference curve to the right, i.e.  $\Delta c_{NL} > 0$  and  $\Delta c_L < 0$ . Under this change,  $\nabla_{\delta} p_L^* < 0$  and  $\nabla_{\delta} p_H^* < 0$  imply that  $\lambda \nabla_{\delta} \pi_L + (1 - \lambda) \nabla_{\delta} \pi_H > 0$ . We have shown that the pooling contract  $(c_{NL}^H, c_L^H) = (c_{NL}^L, c_L^L)$  cannot be a solution to the exogenous problem with  $\bar{p}$  and the pooling contract is  $p$ -distorted upward.

If  $\mu$  is negative, consider a movement of the pooling contract up along the  $L$  type's indifference curve to the left, i.e.,  $\Delta c_{NL} < 0$  and  $\Delta c_L > 0$ . Under this change,  $\nabla_{\delta} p_L^* > 0$  and  $\nabla_{\delta} p_H^* > 0$  imply that  $\lambda \nabla_{\delta} \pi_L + (1 - \lambda) \nabla_{\delta} \pi_H > 0$ . We have shown that the pooling contract  $(c_{NL}^H, c_L^H) = (c_{NL}^L, c_L^L)$  cannot be a solution to the exogenous problem with  $\bar{p}$  and the pooling contract is  $p$ -distorted downward.

We have proved that if  $\bar{p}$  is the optimal average risk in the endogenous solution, then the optimal contracts in the monopoly's endogenous problem are not a solution to the monopoly's exogenous problem with  $\bar{p}$ . Contracts of both types are distorted away from their respective allocations in the exogenous problem. Specifically, the  $H$  type's and  $L$  type's contracts are  $s$ - or  $p$ -distorted upward when  $\mu$  is positive while both are  $s$ - or  $p$ -distorted downward when  $\mu$  is negative.

## E Proof of Lemma 2

We first prove the contrapositive of the forward direction of the claim. Suppose that a leftward movement of the  $H$ -type contract  $(c_{NL}^H, c_L^H)$  from the pooling zero-profit contract  $(c_{NL}, c_L)$  along the  $H$  type's indifference curve  $U^H$  raises profits from  $H$  types. To maintain the zero-profit constraint, profits from  $L$  types must decrease. This implies a movement of the  $L$ -type contract to the right to  $(c_{NL}^L, c_L^L)$ . The new  $L$ -type contract will give  $L$  types higher utility, by the single crossing property. Thus the pooling zero-profit contract  $(c_{NL}, c_L)$  is not the  $U^L$ -maximizing allocation for this particular  $U^H$ .

For values of  $U^H$  where moving the  $H$ -type contract to the left of the pooling zero-profit contract is profitable, the optimal  $H$ -type contract is one that maximizes  $H$  type's profits on the indifference curve  $U^H$ . Graphically, it is at the intersection of the  $H$ -type profit maximization curve and the indifference curve  $U^H$ , where the iso-profit curve is tangent to the indifference curve. The  $U^L$ -maximizing contract is the point on the indifference curve  $U^H$  that yields profits of  $-\frac{1-\lambda}{\lambda}\pi_H$  (for  $\lambda \in (0, 1]$ ) from  $L$  types.

Clearly, this choice of  $L$ -type contract  $(c_{NL}^L, c_L^L)$  and  $H$ -type contract  $(c_{NL}^H, c_L^H)$  satisfies  $(IC_H)$ , monotonicity and the zero-profit constraint. To see why the  $L$ -type contract maximizes  $U^L$ , first observe that the  $L$  type's utility increases from the left to right along the indifference curve  $U^H$  by the single crossing property and that any point to the right of  $(c_{NL}^L, c_L^L)$  results in lower profits from  $L$  types. Therefore for this particular  $H$ -type contract, the  $L$ -type contract  $(c_{NL}^L, c_L^L)$  maximizes  $U^L$ . Moreover, any other  $H$ -type contract on this indifference curve will yield lower profits from  $H$  types and thus raise the required value of profits from  $L$  types. The corresponding  $L$ -type contract will necessarily lie on the left of  $(c_{NL}^L, c_L^L)$  and yield a lower  $U^L$ .

We now prove the backward direction of our previous claim, that the pooling zero-profit contract will maximize the  $L$  type's utility if moving the  $H$ -type contract from the pooling zero-profit contract to the left does not increase profits. With the assumption of single-peakedness of profit along an indifference curve, such a movement will not increase profits when the  $H$  type's profit maximization point lies on the right of the pooling zero-profit contract  $(c_{NL}, c_L)$ .<sup>15</sup> If we pick the  $H$  type's profit maximization point as the  $H$ -type contract, there will be no  $L$ -type contract that yields a zero aggregate profit and satisfies monotonicity at the same time. By similar reasoning, any  $H$ -type contract that lies on the right of  $(c_{NL}, c_L)$  is not feasible. On the other hand, if we move the  $H$ -type contract to the left, the possible candidates for the  $L$ -type contract will necessarily lie on the left of  $(c_{NL}, c_L)$  and yield a lower  $U^L$ . Hence for these values of  $U^H$ , the pooling zero-profit contract maximizes the  $L$  type's utility.

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<sup>15</sup>If the  $H$  type's profit-maximizing contract coincide with the pooling zero-profit contract, then it is clear that the pooling zero-profit contract is the  $U^L$ -maximizing  $L$ -type contract.

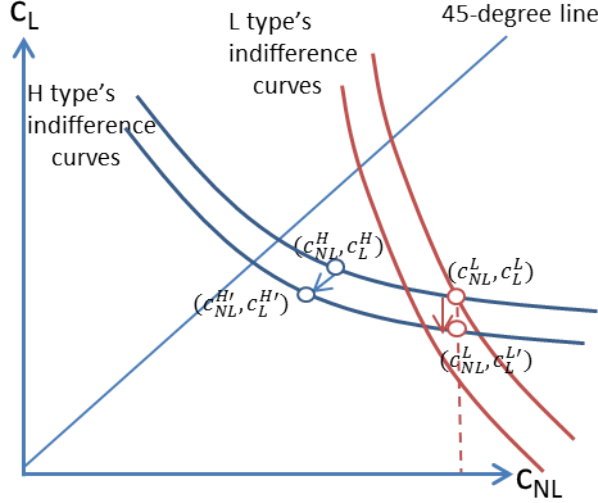


Figure 22: A movement of contracts that improves profits without leaving any type worse off

## F Proof of Lemma 4

Let  $(c_{NL}^L, c_L^L)$  and  $(c_{NL}^H, c_L^H)$  be the *MWS* contracts with endogeneity and  $\bar{p}^{MWS}$  be the associated level of average risk. Let  $U^H$  and  $U^L$  be the indirect utilities of *H* types and *L* types attained at their contracts respectively. Note that  $(c_{NL}^L, c_L^L)$  and  $(c_{NL}^H, c_L^H)$  is a solution to the program  $F$  with  $\bar{p}^{MWS}$  and  $\tilde{U}^H(\bar{p}^{MWS})$ .

Suppose there is a slight decrease in  $\bar{p}$ . Consider a small movement of the *H*-type contract and *L*-type contract to  $(c_{NL}^{H'}, c_L^{H'})$  and  $(c_{NL}^{L'}, c_L^{L'})$  respectively, holding  $U^H$  constant and  $(IC_H)$  binding, such that  $\pi_H(c_{NL}^{H'}, c_L^{H'}) \geq \pi_H(c_{NL}^H, c_L^H)$ . Refer to Figure 22. Note that each type's indifference curve after the decrease in  $\bar{p}$ , holding utility constant, is strictly below the original indifference curve.

We will show that *L* types are not worse off purchasing  $(c_{NL}^{L'}, c_L^{L'})$ . The assumption about  $\eta_{p,\bar{p}}$  allows us to compare the amount of vertical shift of the two indifference curves at the original *L*-type contract. Particularly, the amount of vertical shift of the indifference curve of each type after a decrease in  $\bar{p}$ , holding utility constant is

$$\left| \frac{\frac{\partial U^i}{\partial \bar{p}}}{\frac{\partial U^i}{\partial c_L}} \right| = \left| \frac{(u(c_L^L) - u(c_{NL}^L))}{p^i u'(c_L^L)} \frac{\partial p}{\partial \bar{p}} \right|_h = \left| \frac{(u(c_L^L) - u(c_{NL}^L))}{\bar{p} u'(c_L^L)} \eta_{p,\bar{p}}^i \right|$$

Since  $|\eta_{p,\bar{p}}^H| \leq |\eta_{p,\bar{p}}^L|$  at  $(c_{NL}^L, c_L^L)$ , we have  $\left| \frac{\frac{\partial U^H}{\partial \bar{p}}}{\frac{\partial U^H}{\partial c_L}} \right| \leq \left| \frac{\frac{\partial U^L}{\partial \bar{p}}}{\frac{\partial U^L}{\partial c_L}} \right|$ . This implies that the vertical shift of the *L* type's indifference curve is at least as great as the vertical shift of the *H* type's indifference curve at  $(c_{NL}^L, c_L^L)$  and thus  $U^L(c_{NL}^{L'}, c_L^{L'}) \geq U^L$ .<sup>16</sup>

<sup>16</sup>For the subclass  $h(\frac{p^\alpha}{\bar{p}})$ , the vertical shifts of the indifference curves for both types will be exactly the same. Our results also apply to a related class of cost functions  $h(\frac{(1-p)^\alpha}{1-\bar{p}})$ , which exhibits constant

This new set of contracts yields the same or greater utility for both types and an overall positive profits. Thus  $\frac{dF(\bar{p}^{MWS}, \tilde{U}^H(\bar{p}^{MWS}))}{d\bar{p}} < 0$ . Note that although the optimal *MWS* contracts depicted in Figure 22 are distinct, the proof goes through the case when the contracts are identical.

## G Proof of Proposition 3

Let  $\bar{c}^H, \bar{c}^L$  be the *MWS* contracts with endogeneity and the  $\bar{p}^{MWS}$  be the associated level of average risk in endogenous *MWS* equilibrium.

Since the program *G* differs from *F* by the consistency constraint, we have  $G(\bar{p}, U^H) \leq F(\bar{p}, U^H)$  for any pair of  $(\bar{p}, U^H)$ . The *MWS* allocation  $(\bar{c}^H, \bar{c}^L)$  is an optimal allocation of the program *F* with  $(\bar{p}^{MWS}, \tilde{U}^H(\bar{p}^{MWS}))$  and also satisfies the program *G* with  $(\bar{p}^{MWS}, \tilde{U}^H(\bar{p}^{MWS}))$ , because the consistency constraint is satisfied by the definition of an *MWS* endogenous allocation. We know that  $F(\bar{p}^{MWS}, \tilde{U}^H(\bar{p}^{MWS}))$  and  $G(\bar{p}^{MWS}, \tilde{U}^H(\bar{p}^{MWS}))$  are the *L*-type utilities at the optimal allocations in program *F* and *G* respectively. Therefore  $F(\bar{p}^{MWS}, \tilde{U}^H(\bar{p}^{MWS})) = G(\bar{p}^{MWS}, \tilde{U}^H(\bar{p}^{MWS}))$ .

It follows that the multiplier on the consistency constraint in the program *G* must be zero. If it is nonzero, when the monotonicity constraint does not bind, a movement of the *H*-type contract along the *H* type's indifference curve would change the value of the program because of its effect on the consistency constraint, while leaving the objective and other constraints unchanged. When monotonicity binds and  $(MU_H)$  does not, a movement of the pooling contract along the downward sloping pooled zero-profit curve will change the value of the program because of its effect on the consistency constraint, while leaving the objective and other constraints unchanged.<sup>17</sup>

Thus

$$\frac{dG(\bar{p}^{MWS}, \tilde{U}^H(\bar{p}^{MWS}))}{d\bar{p}} = \frac{dF(\bar{p}^{MWS}, \tilde{U}^H(\bar{p}^{MWS}))}{d\bar{p}} < 0,$$

where the inequality follows from Lemma 4.

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compensated elasticity of  $1 - p$  with respect to  $1 - \bar{p}$ , i.e.,  $\eta_{1-p, 1-\bar{p}} = \frac{1-\bar{p}}{1-p} \frac{\partial(1-p)}{\partial(1-\bar{p})} \Big|_h = a$  for some constant  $a$ . This class of functions guarantees that the magnitude of the horizontal shift of the indifference curve at a point to be the same for both types, which implies a greater vertical shift of the indifference curve at the point for *L* types than for *H* types.

<sup>17</sup>It is not possible for both monotonicity and  $(MU_H)$  to bind at the equilibrium – if both types wish to purchase insurance, then the binding of  $MU_H$  necessarily implies that the contracts are distinct because the type-*i* contract has to lie on type *i*'s iso-profit curve associated with  $\pi_i = 0$ .